A Design Method for Asymmetric and Symmetric, Planar Gradient and Shim Coils

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Abstract
A novel method is presented for designing biplanar gradient and shim coils for use in open MRI systems. The primary coils consist of windings arranged on two parallel planes, located at \( x = \pm a \). There is also a shield set of windings on the planes \( x = \pm b \), with \( b > a \). The method involves a type of target-field approach, in which the desired \( x \)-component \( H_x \) of the magnetic field is specified in advance. Winding patterns on the primary and shield coils are then designed to produce a field that matches the target as closely as possible, while also minimizing the field external to the shields.

Introduction
In conventional MRI systems, the magnets and coils are arranged cylindrically, with the patient lying along the centre. This has certain advantages in terms of the quality of the image that is finally produced. A detailed description of these coils is given by Jin [1]. Possibly the best known method for designing magnets and coils in cylindrical geometry is Turner’s target-field method (see, e.g. [2]). Nevertheless, conventional cylindrical MRI systems can result in a claustrophobic experience for many patients [3]. To overcome this difficulty, magnets with asymmetrically located target zones have been developed by Crozier and Zhao [4], and gradient and shim coils have been designed for such systems by Forbes and Crozier [5]. As an alternative to cylindrical systems, biplanar MRI magnets and coils may be considered. Here, a novel method is presented that explicitly takes account of the exact geometry of the primary and shield plates. The technique is based on a regularized solution of an integral equation that comes from the Biot-Savart law. Coil winding patterns are then generated using a streamfunction method [6].

Methods
The current density vectors on the primary plates and shields are written \( \mathbf{j} = j_y \mathbf{e}_y + j_z \mathbf{e}_z \) (A/m). The Biot-Savart law appropriate to a thin current sheet is then

\[
\mathbf{H}(r) = -\frac{1}{2\pi} \sum_{n=1}^{N} \sum_{m=1}^{M} \frac{B}{n\pi} \sin \left( \frac{n\pi (y' + L)}{2L} \right) \sin \left( \frac{n\pi (z' + B)}{2B} \right) \frac{1}{|r - r'|} d\Lambda',
\]

in which the magnetic field vector is \( \mathbf{H}(r) = H_x \mathbf{e}_x + H_y \mathbf{e}_y + H_z \mathbf{e}_z \) (A/m). The location of a field point (interior or exterior to the coil) is \( r \), and \( r' \) is the vector position of a source point on the primary or shield plates. The sum is taken over all primaries or shields in the system. The two components of the current density \( \mathbf{j} \) on each plate can be represented in terms of a streamfunction \( \psi \), which follows from the continuity equation on each plate. It can be shown that

\[
\psi(y', z') = \sum_{n=1}^{N} \sum_{m=1}^{M} P_{nm} \sin \left( \frac{n\pi (y' + L)}{2L} \right) \sin \left( \frac{n\pi (z' + B)}{2B} \right) + j_y = \frac{\partial \psi}{\partial z'} \quad \text{and} \quad j_z = -\frac{\partial \psi}{\partial y'}, \quad -L \leq y' \leq L, \quad -B \leq z' \leq B.
\]

Suppose that the target field on some interior or exterior pair of planes \( x = \pm c \) is denoted \( H_T(c, y, z) \). The sets of coefficients \( P_{nm} \) on each plane set are found by minimizing a regularized least-squares type error expression of the form

\[
G = \sum \iint \left[ H_T(c, y, z) - H_x(c, y, z) \right]^2 dydz + \sum \lambda \iint \left| \nabla^2 \psi \right|^2 dydz,
\]

where the sums are taken over all the primary and shield planes. The constants \( \lambda \) are essentially Lagrange multipliers; they are chosen to be small enough to give a close match between the calculated and target fields, but large enough to give a well conditioned system [7].

Results
This technique has been used to design winding patterns for a range of shielded and unshielded coils, in which the target region can be located asymmetrically with respect to the primary plates. An example is given below, for the T11 field \( H_x(x, y, z) = Cx \), with the target region located highly asymmetrically toward the bottom of the diagrams. Winding patterns for the primary and the shield are given in Figures 1(a) and (b), and positive and negative winding directions are indicated. The resulting field is shown in Figure 2, and the dashed lines indicate the locations of (two) interior target zones and the exterior target region on which the field is made to be as small as possible. The success of the design is able to be seen from the high degree of linearity of the field within the interior target zones.

Figure 1(a). Primary winding: asymmetric T11. Figure 1(b). Shield winding: asymmetric T11. Figure 2. HX field on centre-plane \( y = 0 \).

Conclusion
Accurate winding patterns for asymmetric biplanar gradient and shim coils may be designed using this regularized integral equation technique.

References