Gradient Coil Design Using Quadratic Programming with Relaxed Target Field

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Introduction

Today Magnetic Resonance Imaging (MRI) technique require high performance gradient coils. This performance is determined by the compromise efficiency(η [T/mA])-inductance(L [H])-gradient uniformity. Different methods have been presented in the direct and inverse space in order to obtain the best tradeoff [1]. The functional energy minimization scheme presented by Turner [1], produces gradient coils with high efficiency-inductance relation but with worse gradient uniformity. Moreover, is not possible with this method to define the optimal coil length that produces the maximum coil quality. The “universal law” presented by B. Zhang et al. [2] for unshielded gradient coils does not take into account the dependency between coil length and gradient uniformity. H. Xu et al. take into account these dependency but for the main magnet design [3]. The purpose of this paper is:

- To describe a method for gradient coils design able to control the target gradient uniformity.
- To obtain a new coil length law that relates the radius (c) of region of interest (RI), coil radius and the desired uniformity.
- To obtain an unshielded transverse gradient coils with high efficiency-inductance-gradient uniformity compromise, for our build-home low field MRI machine Giroimag [4].

Methods

The method assumes a cylindrical surface current density distribution expressed in Fourier series [5]. The current density is confined on the interval (LC/2≤z≤LC/2), where LC is the coil length and R is the cylinder radius. The stored magnetic energy produced by the current density calculated in the reciprocal space can be written as:

\[ E = \frac{1}{2} \sum_{n=1}^{N} j_n \cdot E_m \cdot j_n \]

where \( N \) is the total number of axial modes of current density, \( j_n \) is the amplitude of each oscillating mode of the current, \( E_m \) is a symmetric matrix defined for longitudinal and transverse gradient coils [1]. As different from other methods [1,5,6], in our method, the magnetic gradient field contribution is obtained from the Biot-Savart law in r space in order to make accurate calculations. From equation (1), the design could be stated as a quadratic programming problem with linear constrains as follows:

\[
\min \frac{1}{2} \sum_{n=1}^{N} G_{m(n)} \cdot j_n \quad \text{such that:} \quad \sum_{n=1}^{N} G_{m(n)} \cdot j_n \leq G_0 (1-\epsilon),
\]

where \( G_0 \) is the desired gradient value, \( \epsilon \) is a desired error uniformity whose peak-peak relative deviation from \( G_0 \) value could be between 1% to 10% or more. In that way, the boundary condition over the surface of RI is relaxed and more practical solutions can be found. The matrix \( G_{m(n)} \) is the gradient field contribution at the target point \( t \) of each axial mode \( n \) of unity amplitude. (x) means \( \frac{\partial B_z}{\partial x} \) and (z) means \( \frac{\partial B_z}{\partial z} \).

Results and Discussion

From the problem defined in (2), it is realized that the energy calculation is independent from the linear gradient field constrains. This statement avoids the apodization step, needed in other methods. Applying our procedure, taking as measure of quality the figure of merit \( \Gamma = \eta [T/mA] / \sqrt{L[H]} \) [5], a linear relations for the optimal \( LC \) that produces maximal figure \( \Gamma \) is obtained. As can be seen from fig.1 more uniform gradient needs longer coil. For the same desired gradient uniformity, the transverse gradient coil must be longer than the longitudinal gradient coil. The method was applied to the design a transverse gradient coil of \( R = 0.325 \) m, radius of RI \( c = 0.2 \) m, with \( \epsilon = 6.2 \)%. Four target points with values \( G_n = 10^{-3} T/m \), were specified over RI. The rms gradient uniformity evaluated with Biot-Savart law was 3.04 % in the RI, the coil inductance \( L_{c} = 535.3 \) µH, while its resistance was 0.31 Ω using circular conductor of 3 mm in diameter. The efficiency is \( \eta = 0.108 \) mT/m and \( \Gamma = 46.6 \times 10^{-4} \). The gradient coil efficiency and inductance values of our design (Fig. 2) are similar to the characteristics obtained with the constrained length design with only field constrains [6], however, the gradient uniformity produced by our design is superior.

Conclusion

A new methodology with control over gradient uniformity has been presented for unshielded gradient coil design. General linear relation has been obtained for the optimum coil length that produces the maximal figure \( \Gamma \). The new methodology was applied to the design of transverse gradient coils for a low field MRI machine.

References