Group Theoretical Approach to rf Coil Design

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Overview

A group-theoretic method is presented for finding closed-form solutions for the normal mode frequencies for coils possessing at least a modicum of symmetry. The coil symmetry allows one to block-diagonalize Kirchhoff's laws. One may then find the normal modes and their frequencies for each block independently. For many coils of interest in MR applications, the frequencies can therefore be expressed in closed form. Such solutions allow one to consider design questions about mode degeneracy of particular relevance to the parallel imaging community. Also, recent work has shown the value of having such solutions in developing coil 'simulators.'

Theory

The ability to find closed-form solutions for the normal mode frequencies of an rf circuit is of value as one develops simulators for coil design. [1, 2] Finding the normal mode frequencies of the rf coil system involves solving the roots of N^{th} order polynomials where N in the number of independent loops in the system. For coils with some degree of symmetry, it is possible to use group theory to block-diagonalize Kirchhoff's laws. With sufficient symmetry, the blocks may be small enough to find closed-form solutions for all the frequencies. The ranks of the blocks are given by the ranks of the irreducible representations of the group, and in practice have been determined to be at most three or four for coils with a moderate degree of symmetry. The block-diagonalization process is writing down a unitary transformation that transforms the system from a basis in which it is easy and convenient to write Kirchhoff's laws to a basis in which it is easy to solve for the normal mode frequencies of the system.

Pedagogical Example

For the system depicted in Fig. 1, it is easy to write down Kirchhoff's law and verify that the unitary matrix,

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

diagonalizes the impedance matrix. For this system one may have guessed this unitary matrix. However, for more complicated systems the formalism of group theory provides a systematic method for constructing such a unitary matrix. This involves finding the projection of the original basis vectors on the block-diagonal basis. The N-dimensional regular representation of the group is reduced to its irreducible parts. This involves finding the character table for the symmetry group and then projecting the original basis vectors onto the irreducible basis vectors.

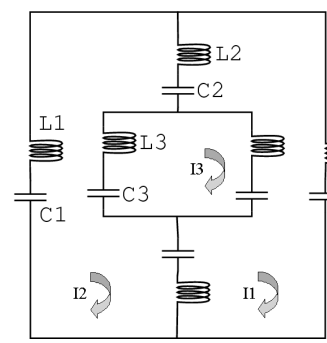

Figure 1: A simple coil with 180° rotational symmetry

Circulant

We note that the solutions presented in [3, 2] for circulant matrices could be obtained by using the group C_n to block-diagonalize Kirchhoff's laws for a birdcage. For systems with circulant symmetry, the block-diagonalization results in N 1x1 blocks or said another way a complete diagonalization. This can be easily understood by the fact that the unitary matrix produced by the group theory formalism is in fact the matrix of eigenvectors for a circulant matrix.

Other Examples

It is possible to find closed-form solutions using this technique for systems for which these solutions were not readily apparent using other methods. One example of such a coil was the three-ring birdcage, first suggested by Jevtic in [4] and discussed more recently in [6]. The results for the frequencies calculated using the group theory approach were in excellent agreement with the experimental measurements. In addition to the aforementioned coil, this analysis was applied to the coil presented by Duensing *et al*. [5] Although closed-form solutions could be arrived at via other means for simple coils, the group theory approach provides a method sufficiently robust so solutions in more complicated cases can be found.

Design Tool

The symmetry of the coil may dictate certain modes having degenerate frequencies; for instance, the birdcage symmetry leads to a minimum of N/2-1 pairs of degenerate frequencies. (This degeneracy is the key for using the birdcage in quadrature receive mode.) Using the tools available from group theory, one may be able to ask that instead of pairs there should be, say, triplets of degenerate modes. This condition would then dictate the rank of the irreducible representations of the group to three. One now decides which group satisfies this condition, and the symmetry of the coil, and hence its design, is thus determined.

References

[1] C.-L. Chin, C. Collins, S. Li, B. Dardzinski, and M. Smith. Birdcagebuilder: Design of specified-geometry birdcage coils with desired current pattern and resonant frequency. *Concepts in Magnetic Resonance*, 15(2):156-163, 2002.

[2] G. Giovannetti, L. Landini, M. Santarelli, and V. Positano. A fast and accurate simulator for the design of birdcage coils in MRI. *Magnetic Resonance Materials in Biology, Physics and Medicine*, 15(1-3):36-44, 2002.

[3] M. Leifer. Resonance modes of the birdcage coil. *J. Magn. Reson*., pages 51-60, 1997.

[4] J. Jevtic. Ladder networks for capacitive decoupling in phased-array coils. In *Proceedings of the ISMRM ninth scientific meeting and exhibition, Glasgow, Scotland, UK, April 1-7, 2001*, page 17. International Society for Magnetic Resonance in Medicine, 2001.

[5] G. R. Duensing, D. M. Peterson, B. Beck, and J. R. Fitzsimmons. Transmission field profiles for transceive phased array coils. *In Proceedings of the ISMRM Eighth Scientific Meeting and Exhibition, Denver, Colorado, USA, April 1-7, 2000* International Society for Magnetic Resonance in Medicine, 2000

[6] T. Eagan, T. Chmielewski, J. Flock, Y. Cheng, T. Kidane, Sh. Shvartsman, G. DeMeester, W. Dannels, and R. Brown. Understanding approximately degenerate eigenmodes in two-ring and three-ring RF birdcages. In *Proceedings of the ISMRM tenth scientific meeting and exhibition, Honolulu, Hawaii, USA, May 18-24, 2002*, Honolulu, Hawaii, USA, 2002. International Society for Magnetic Resonance in Medicine, 2002 .