Equi-Ripple Design of Quadratic Phase RF Pulses

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Quadratic phase RF pulses with high selectivity and broad bandwidths are developed using the Shinnar-Le Roux transformation. Unlike previous implementations, the required quadratic phase FIR filters are designed using the complex Remez exchange algorithm, which ensures an equi-ripple error function without the need for heuristic spectral weighting. It is argued analytically that these pulses yield near-optimal solutions in terms of minimising the B_1-amplitude. The effectiveness of typical pulses is demonstrated *in vitro* and *in vivo*.

Introduction

High selectivity and broad bandwidth are desirable goals in radio-frequency (RF) pulse design. In the linear phase regime, such requirements lead to long RF pulses with high maximum B_1. For saturation and inversion purposes, these limitations can be overcome by abandoning the linear phase and rather generating a quadratic phase, as shown by Le Roux et al. [1]. The design of linear phase pulses is commonly performed by creating an equi-ripple FIR filter, followed by the inverse Shinnar-Le Roux (SLR) transformation [2,3]. However, the design problem for quadratic phase FIR-filters is considerably more complex. Le Roux originally proposed a weighted least squares (WLS) procedure, using a heuristic weighting function to approximate an equi-ripple solution. In the present work, we propose an alternative approach based on the complex extension of the Remez exchange algorithm [4]. The new approach yields truly equi-ripple solutions without further considerations and performs robustly for a wide range of quadratic target phases.
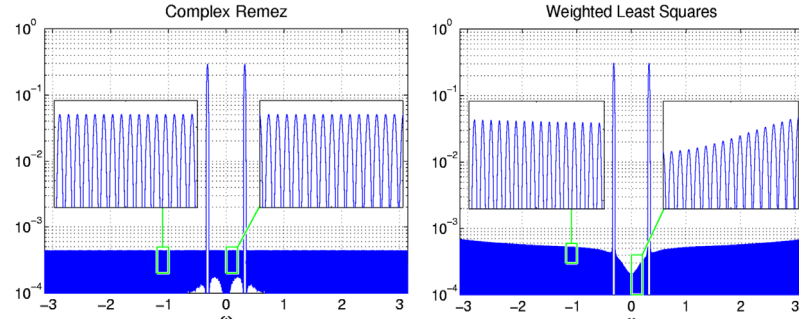

Fig.1: Error function for FIR filters obtained with complex Remez and WLS (n=512, θ=90°, ω_p=0.3, ω_s=0.35, k=150). Windows in the stop and pass bands are magnified to reveal ripple structure. The overshoots lie in the transition bands and do not reflect actual errors.
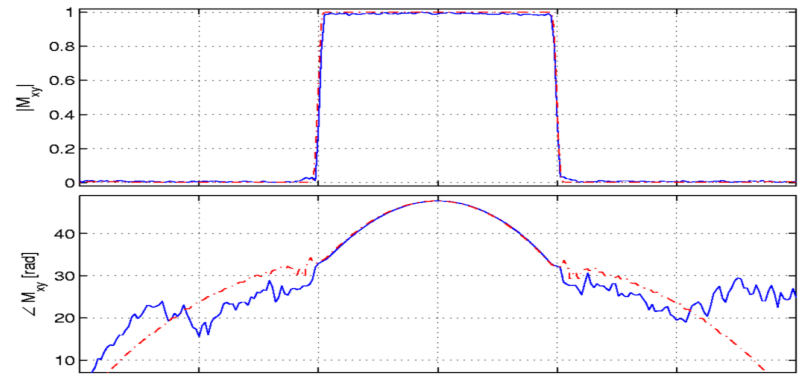

Fig.2: Excitation (M_{xy}) profiles simulated with Runge-Kutta (dash-dotted) and measured (solid) (using same Remez pulse as in Fig.1). The magnitude is shown at the top. Note the exact realisation of the quadratic phase, as depicted below.
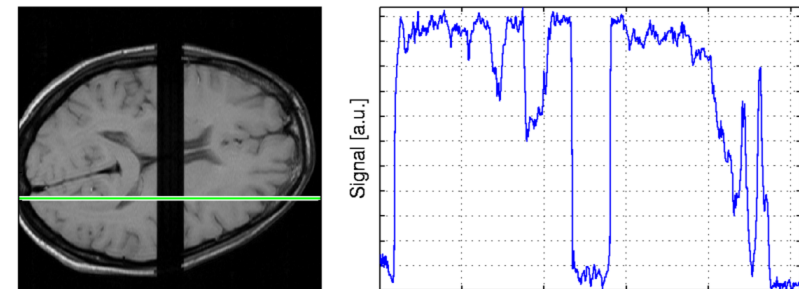

Fig.3: Highly selective saturation by a quadratic phase pulse exemplified *in vivo* in a healthy volunteer (using same Remez pulse as in Fig.1). A profile along the horizontal line on the left is plotted on the right.

Theory and Methods

Linear phase slice selection pulses are essentially sinc-like, with long duration and a high maximum B_1 field. To reduce this maximum, the pulse power must be spread across the whole pulse duration. This precisely is achieved with a quadratic phase. Qualitatively, this behaviour can be explained through the small tip angle approximation. For a sufficiently large amount of quadratic phase k, the following functions form a Fourier pair [5]: $B_1(t)=\gamma|B_1|$ rect$(t/2k)$ exp$(-it^2/4k)$, $M_{xy}(\omega)\approx\sqrt{(-4\pi ki)}$ exp$(ik\omega^2)$ rect(ω), where rect denotes a boxcar function. That is, a rectangular slice profile can be excited with an RF pulse with finite duration and constant B_1 amplitude. Note that the Fourier transformation preserves power. Hence, all pulses with a rectangular profile and the same bandwidth deposit the same power. A constant $|B_1|$ with sufficient quadratic phase therefore minimises B_1^{max} for any given bandwidth.

Quadratic phase FIR filters are characterised by the filter length n, the flip angle θ, the pass and stop band frequencies ω_p and ω_s and the amount of quadratic phase k. A corresponding target function is specified for fitting the polynomial coefficients of the FIR filter. k needs to be sufficiently large in order to spread out B_1 power, while not larger than a critical value that depends on ω_p and ω_s.

The Remez exchange algorithm identifies the best polynomial fit by minimising the Chebyshev (maximum) error norm. Along with the complex Remez algorithm, a WLS procedure was implemented for comparison. Once the desired filter is obtained, a complementary minimum-phase A-polynomial is generated for the SLR transform to produce the RF pulse [2].

The generated RF profiles were verified by a numerical integration with the fourth-order Runge-Kutta method. Furthermore, they were implemented on a Philips 1.5T Intera whole body scanner equipped with a transmit/receive birdcage resonator (Philips Medical Systems, Best, The Netherlands), as both excitation and suppression pulses.

Results

Figure 1 shows typical results of FIR filter design with the complex Remez and WLS algorithm (magnitude of desired minus actual filter response). The Remez algorithm yields an equiripple solution, while the WLS algorithm would require some tuning of the weighting function. Figures 2 and 3, respectively, show the excitation and suppression profiles achieved experimentally with a typical Remez pulse.

Discussion/Conclusion

This work shows that the complex Remez exchange algorithm is a powerful method for the flexible design of quadratic phase pulses. The quality of the resulting pulses has been demonstrated by experimentally obtained profiles. These pulses can be further optimised by adjusting the weighting of the pass and stop bands during design procedure. Both design methods yield qualitatively similar results with the ability to individually specify an arbitrary amount of quadratic phase.

References

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