Non-Orthogonal Non-Fourier Spatial Encoding for Dynamic MRI

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Introduction

In MRI, spatial encoding almost exclusively utilizes orthogonal basis sets of encodes. This is mainly due to the straightforward nature of reconstruction - requiring the inverse of the encoding transform - that fortuitously can use the equivalent conjugate transpose. Fourier encoded MRI and non-Fourier methods such as wavelet MRI, Hadamard MRI, and SVD-encoded MRI all represent applications of orthogonal basis sets for spatial encoding. A notable exception is the pseudo-Fourier imaging method of Kadah and Hu (1) who introduced data acquisition using the non-orthogonal Gabor transform functions. Kadah and Hu described their approach using the mathematics of frames, a formalism designed for creating representations based upon a typically large set of non-orthogonal functions.

We introduce a new general formalism for data acquisition employing non-orthogonal non-Fourier spatial encoding during dynamic MRI derived partly from the theory of our earlier approach for non-Fourier fluoroscopic MRI (2,3). This general approach subsumes the methods of Fourier (4,5) and non-Fourier fluoroscopic MRI.

Our approach uses two major concepts: first, the image contribution of the set of non-orthogonal encodes is viewed solely as its projection onto a chosen orthogonal basis that sufficiently encodes the FOV contents; and, second, reconstruction is performed iteratively, using the principle of an enforced self-consistency between the acquired data and computed estimates of the acquired data to accomplish an advantageous inverse-free reconstruction algorithm. Our approach also provides for fluoroscopic reconstruction with incomplete data, i.e. streaming reconstruction and display from small subsets of encodes. Properties are ideal for adaptively imaging select regions-of-interest of the FOV useful for interventional MRI.

Methods

Our method in 2D is based upon (a) the matrix description of the discretized k-space associated with the FOV, and (b) its analytic description in terms of the portions encoded by the non-orthogonal encodes used for acquisition projected onto a conventional orthogonal basis that completely encodes the FOV contents, and (c) the estimated complete acquisition less this non-orthogonally encoded term. If low flip angle spatial selective excitations are used, acquired samples from the FOV represent $Saq = U^*Kfov$ where Saq are the low-wise acquired samples and $U^*$ is the conjugate transpose of an orthogonal vertically-encoding basis set and Kfov is the total k-space matrix acquired from the FOV (frequency encoding in the horizontal direction is assumed). Expanding $U^*$ to add/subtract a term representing the projection of a non-orthogonal encode set, G, onto the U basis, and reconstructing to a k-space representation, gives our fundamental equation: $Krecon = U^*\left( UU^*G + a\right) Kest + (UU^*G)Kacq$ (* implies conjugate transpose). Here $Kacq = G^*Kfov$ is the actual non-orthogonal encoded sampled data, and $Kest = Kfov$ is substituted into the first term to initially utilize the information from a good estimate of the FOV, eg. data acquired from a previous image. Thereafter, in a series of reconstruction computation iterations, Kest is set to Krecon. Applying a numerical relaxation factor gives the computational working formula:

$Krecon = Pu Kest + a Pu (b G^*Kacq - Pg Kest)$  

where $a$ is a 0-1 relaxation factor, $b = ||PgKest|| / ||G^*Kacq||$ a normalization factor, $Pu =UU^*$ is the projection operator that projects onto a known orthogonal basis, and $Pg = GG^*$ is the operator projecting onto the non-orthogonal encode set. This method provides a rigorous basis such that data acquired from non-orthogonal encodes can be added to known FOV k-space information with the subtraction of the corresponding estimated portion to compensate.

Using encodes described in columns of G, samples in Kacq are acquired by the scanner. Reconstruction involves iteration through Eq.1 above. (Note that Kacq can represent as little as a single sample during a dynamic fluoroscopic MRI series. Also, encodes in G may encode only a small portion of the FOV if desired or mix encodes from orthogonal and non-orthogonal encode sets.) First, $Kest = Kfov$ is substituted into the first RHS term. Thereafter, computation of Krecon uses $Kest = Krecon$ from the previous iteration. Self consistency is enforced after approx 10-20 iterations for reconstruction, noting that the reconstructed k-space, Krecon, is obtained without any inversions necessary. Krecon k-space data is inverse Fourier transformed to obtain the magnitude image for display.

Results

Image data were acquired from a normal adult volunteer on a 1.5T GE SIGNA LX Echospeed MR scanner (GE Med.Sys., Milwaukee WI) using a standard T1-weighted spin echo pulse sequence (TE 20ms, TR 800ms, 1NEX, FOV 24cm, 10mm thick) adapted for low flip angle non-Fourier spatial encoding, using an initial spatially selective alpha excitation pulse and a slice selective 180deg pulse. Data were acquired using sets of 8, 16 and 24 RF pulses representing Gaussian, Lorentzian and sinc-shaped excitation spatial domain profiles (non-orthogonal sets) evenly distributed in the FOV for horizontal encoding.

Fig.1 displays a study of the mixture of information from 8 Gaussian encodes (s=1.5cm) and the known k-space acquired earlier from a normal adult brain. Fig.1 left displays a naively reconstructed result using the Moore-Penrose pseudo-inverse of the non-orthogonal Gaussian basis; middle is the result reconstructed using our approach, showing more structure and detail; and, right is the standard RARE slice used initially as Kest.

Conclusions and Discussion

We introduce a new formal approach for acquiring dynamic MRI data using a non-orthogonal set of encodes that involves a method for inversion-less reconstruction, an advantage when pseudo-inverse reconstruction is inappropriate or impossible due to ill-conditioning and/or singularities. The method provides an avenue for free-form streaming MRI data acquisition that generalizes the original concept of Riederer's fluoroscopic MRI (4,5) and our approach for non-Fourier MR fluoroscopy (2,3). Its use extends to mixed sets of encodes, fluoroscopy using one or only a few encodes (a major departure from the frames approach), and, most importantly, is a formalism for acquiring non-orthogonal non-Fourier encodes for 3D dynamic MRI. The method provides a significant and extremely flexible MRI data acquisition technique that can contribute to dynamic imaging tools available for applications such as interventional MRI when streaming acquisition may employ highly unconventional shaped RF.

References