Optimal Sampling for 3D Projection Reconstruction Imaging

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Introduction

Projection reconstruction (PR) has received attention recently for several reasons: (1) it uses very short echo times (TEs), (2) spatial resolution is determined by readout resolution, and (3) it significantly oversamples the center of k-space. Short TE allows for rapid acquisition, imaging short T2 species and reduced motion and flow artifacts. All regions inside the radius satisfying the Nyquist criterion are oversampled with the area around the origin being most oversampled. Oversampling the region near the origin is advantageous because this area contains most of the energy in k-space. Investigators have shown that in 2D angular undersampling by a factor of up to sixteen times yields acceptable results [1]. 2D acquisition planes have been stacked to form a pseudo-3D PR sequence, with acceptable results found undersampling by factors up to eight [2]. Recently, 3D PR was demonstrated using VIPR where undersample rates up to 100 might be possible [3]. In this paper, 2D, psuedo-3D and 3D k-space trajectories (Figure 1) are compared in terms of alias free FOV.

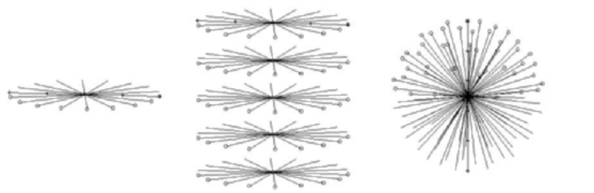

Figure 1. PR trajectories circles indicate end of projections and * indicate samples along one projection. Left: 2D PR. Center: Pseudo-3D PR trajectory of stacked 2D acquisitions. Right: 3D PR trajectory using icosahedron-based angular directions.

Theory

The Nyquist criteria for 3D PR acquisition can be expressed in terms of the distance between radial samples (Δk_r) and distance between adjacent angular samples (Δk_a), $\Delta k_{amax} \leq \Delta k_r$. However, in the interest of minimizing scan time and artifacts, the angular sampling directions should be uniformly distributed on the surface of the spherical sampled volume. $\Delta k_{amin} = \Delta k_{amax}$ between adjacent rays. The exact distance between adjacent samples for a uniform distribution d, can be found by assuming that three neighboring points form an equilateral triangle and that each point is the corner of six such triangles [4]. The surface area of one such equilateral triangle radially projected on the surface of the unit sphere is A. Each sample point possesses A/3 of each triangle in which it participates, and being part of six triangles apportions it a total area of 2A. The total surface area of the unit sphere 4π is thus divided into N hexagons each of area 2A.

$$N(d) = \frac{2\pi}{A(d)} = \frac{\pi}{\operatorname{asin}\left(\frac{d^2\sqrt{3-d^2}}{(4-d^2)^{3/2}}\right)}$$

3D PR scan times become excessively long if enough projections are acquired to meet the requirements of this Nyquist criterion (Nr=256, Np = 237739, TR = 10 ms, scan time ~40 min.). However, a significant advantage of PR is that k-space is oversampled inside the radius where the Nyquist criterion is met. As PR sampling schemes are progressively more undersampled, an alias free field of view (FOV) is retained. Specifically, the diameter of the alias free FOV is propotional to $Np^{1/2}$ for 3D, to $Np^{2/3}$ for stacked 2D and to Np for 2D (Figure 2). A sphere of equal volume to the stacked 2D FOV is used to determine the diameter for stacked 2D.

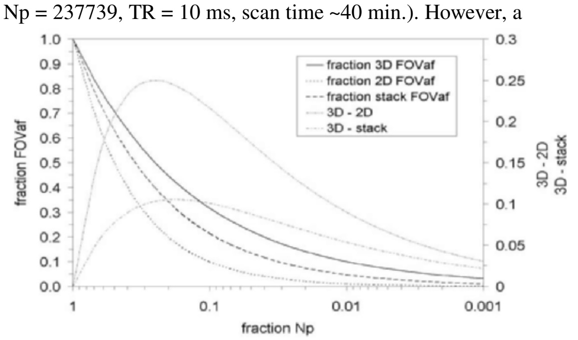

Figure 2. Alias free FOV is plotted as a fraction of original FOV versus the number of projections (Np) as a fraction of the original Np determined by the Nyquist criterion. The second y-axis on the right is the scale for the difference between 3D and 2D or stacked 2D.

Several approaches have been developed to find the locations of uniformly distributed points on the surface of a sphere. The uniformity of two of these approaches has been verified with respect to the equation N(d), numerically solving for the minimum energy positions of point charges on a perfectly conducting sphere [5], and icosahedron-based pixelization scheme [6] (Figure 3). As might be expected, the expression for N(d) is not exact for d > 0.7 radians (N ≤ 20).

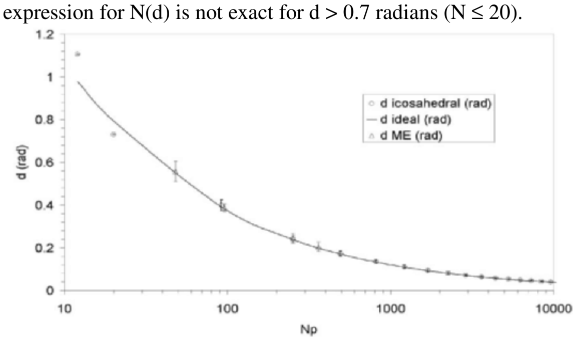

Figure 3. The distance (d) between neighboring points on the surface of the unit sphere versus the number of points or projections (Np). Line represents the equation N(d), points represent mean immediate neighbor distance ± one standard deviation. Circles represent icosahedron-based directions and triangles represent minimum energy-based directions.

Results

Point response function for 3D PR using an icosahedron-based projection direction set was simulated. When compared with the 2D spoke pattern the artifact due to undersampling for 3D looks similar while less intense. Using undersampling rate an order of magnitude larger for 3D than 2D PR still results in a larger alias free FOV in 3D (Figure 4).

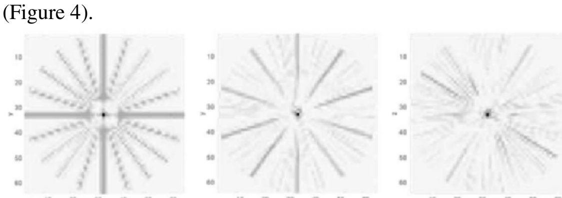

Figure 4. Point response function comparison. Left: 10x undersampled 2D PR. Center and Right: 118x undersampled 3D PR central x/y plane (z=32), central y/z plane (x=32). Image dimensions are 64x64 and 64x64x64.

Discussion

In this study a sampling criterion for 3D PR has been developed. The application of this criterion and uniform angular sampling were used to study the relationship between undersample rate and alias free FOV for 3D, stacked 2D and 2D PR. In general, 3D PR is much less sensitive than 2D PR to the angular undersampling artifact.

Acknowledgments

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References

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