

Noise Removal with Tissue Boundary Preservation Using Fourth-Order Partial Differential Equations

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Introduction

A major source of MR image degradation, with impairment of diagnostic information, is random thermal noise entering the MR data in the time domain [1]. Methods for noise suppression are thus an important issue in many MRI applications. To overcome the deficiencies of acquisition-based noise reduction methods, such as increased acquisition time (i.e. time averaging over repeated measurements) or decreased spatial resolution (i.e. enlarging voxel volume), several postprocessing methods have been proposed. The ultimate goal of these methods is to obtain piecewise constant, or slowly varying signals in homogeneous tissue regions while preserving the tissue boundaries. The most powerful and promising mathematical approaches towards this goal are PDE-based methods (e.g. [2],[3],[4]) and wavelet-based methods (e.g. [5]).

Methods

PDE model: We use a partial differential equation of 4th order to remove noise from MR images. It is assumed that the noise level is approximate known. We are trying to get a new image u that has the same noise level but with a smallest norm for the second order derivatives, i.e. we are trying to minimize the norm of the second order derivatives under the noise level constraint. The minimization problem is solved by a Lagrangian technique in connection with finite difference approximations. More specifically, we assume u_d is the image to be restored. Then we update u^k as in the following with an appropriate u_0 .

$$u^{k+1} = u^k - dt \left[\left(\frac{u^k_{xx}}{|u^k_{xx}|} \right)_{xx} + \left(\frac{u^k_{yy}}{|u^k_{yy}|} \right)_{yy} + \left(\frac{u^k_{zz}}{|u^k_{zz}|} \right)_{zz} + \lambda^k (u^k - u_d) \right]$$

As k gets large, u^k should approach the minimizer. Numerical experiment indicates that $dt = 0.2$ is a good step size. λ is updated using a project method as in [3].

Imaging data: We tested our algorithm on two data sets. Both were acquired on a Siemens Vision 1.5 T at DKFZ in Heidelberg for the European COST B11 concerted action on quantitation of MR image texture. The first data set is an isotropic 1x1x1mm³ FLASH 3-D head acquisition of a healthy volunteer (fl3d_6b195.wkc, TR=20ms, TE=6ms, FA=30deg, matrix=256x256, 32 slices, with 1, 2, and 4 acquisitions (AC), respectively). The second data set is from a phantom consisting of a pair of tubes filled with plastic straws of two different diameters embedded in doped agarose gels with tissue equivalent T1 and T2 relaxation times. This straws phantom was imaged with SE (TR=150ms, TE=15ms, FOV=75x150mm², matrix=256x512, thick=2mm, 5 slices). The SE pulse sequence generated 8 different measurements together with 7 averages from measurements 1+2, 1+2+3,..., 1+2+...+8, respectively, before magnitude images were calculated (cf. [6]). In each case we applied the restoration algorithm to the series with poorest SNR (i.e. AC=1). The results were compared with the observed average of 4 FLASH 3-D (head) and 8 SE (straws phantom) measurements, respectively, where these averages were taken to represent the "ideal" image.

Results

Results from the 3-D head acquisitions (detail from slice #17) are shown in Fig. 1 [Upper left: original input data (AC=1), Upper middle: result from processing the AC=1 data, Upper right: "ideal" image (AC=4); Lower row: the corresponding histograms for the entire slice]. Notice the shape similarity between the histograms of the restored image (middle) and the "ideal" image (right). In this experiment, 26 iterations on the full 3-D data set consumed less than 13 minutes on a notebook computer equipped with a 500 MHz Pentium III. The effect of PDE noise removal in a single measurement from the straws phantom (detail from slice #1) is depicted in Fig. 2 [Upper row: small diameter straws, Lower row: large diameter straws; Left column:

original input data (AC=1), Middle column: results from processing the AC=1 data, Right column: "ideal" image (AC=8)]. Notice the smoothing effect in the straw lumen with good preservation of the thin walls.

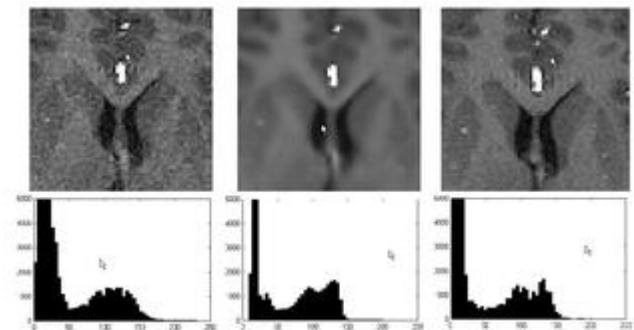


Figure 1. Head acquisition (see text for explanation)

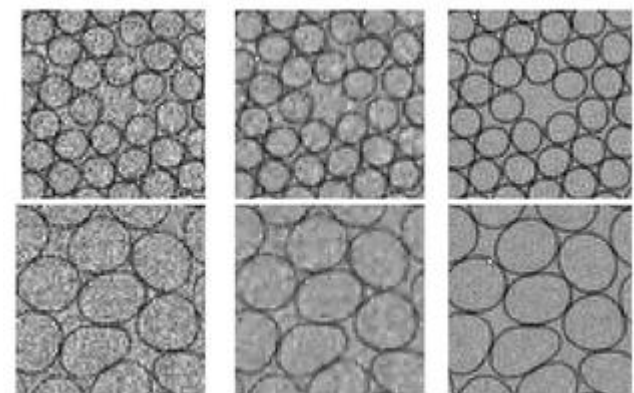


Figure 2. Straws phantom - detail (see text for explanation)

Discussion

A PDE-based noise removal algorithm is presented and tested. The algorithm is reasonably fast (even in Matlab implementation), and fully automated. We are currently exploring the possibility to compute the noise level directly from the input data. The method is adopted to multi-slice 2-D images, true 3-D acquisitions, and to dynamic 3-D + time images. Application in multispectral tissue classification and functional MRI is under investigation aiming at more precise morphometry, improved sensitivity and specificity in BOLD fMRI, and at better estimates of rCBF, rCBV and TTP in perfusion imaging.

References

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