Pressure/Modulus Inversion for MR Elastography

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Introduction Tissue contrast based on elastic properties has significant potential in imaging for breast cancer and other disease [1]. Since MR imaging is not directly sensitive to tissue elastic properties, quantities such as shear modulus must be calculated from measurements of displacement.

In incompressible tissues, the scalar pressure field is an additional unknown quantity which has spatial variation. Knowledge of the pressure field, combined with the elastic modulus, would permit a stress analysis of the tissue in question. From a biomechancial perspective, knowledge of stress and force is very important. In this work, simulations are presented which illustrate the potential for simultaneous inversion of pressure and elastic modulus from a known displacement field.

Methods The vector equation describing static equilibrium in a linearly elastic material is:

$$\nabla \mathrm{p} + \nabla(\mu \nabla \mathbf{u}) = \mathbf{0} \quad (1)$$

where p is scalar pressure, μ is the elastic modulus, and $\mathbf{u}$ is the vector displacement field that is measured with phase contrast MRI. Pressure may be eliminated analytically from these equations with the use of a third differential operation as shown by [2]. However, we retain p as an unknown, such that derivatives of the displacement vector field are limited to second order; this may potentially improve the conditioning of the inversion matrix.

If pressure and modulus are both available, then the stress analysis of the tissue can be performed according to:

$$\mathbf{T} = \mathrm{p}\mathbf{I} + 2\mu\mathbf{E} \quad (2)$$

where $\mathbf{T}$ and $\mathbf{E}$ are the stress and strain tensors respectively.

A simulated two-dimensional displacement field was created for a Gaussian shaped object contained in a square FOV. Normally distributed random values were added to simulate a maximum SNR of 200 in the displacement data. Low pass filtering by sinc convolution was applied to smooth the effects of noise prior to inversion. The boundary conditions for the inversion were assigned to be constant pressure and constant elastic modulus. The solution of equation 1 was performed with a finite difference numerical method.

Results Inverted pressure and modulus are plotted in figures 1a-b, and may be compared to the true values shown in c-d. In graphs e-f, line profiles through the centre of the above images are plotted to demonstrate quantitatively the performance of the inversion. These results indicate that the quality of the modulus reconstruction is similar to that for pressure. However, the pressure reconstruction is more sensitive to noise as SNR is reduced.

Conclusions We have demonstrated with simulations that pressure can be retained as an unknown variable and solved for along with elastic modulus in a linear inversion procedure. Retaining pressure as an unknown has two potential advantages: a third spatial derivative of the displacement data is unnecessary, and a stress analysis of the tissue may subsequently be performed if desired.

The combined pressure/modulus inversion is still somewhat ill-conditioned since noise perturbations to the raw displacement data on the order of 10^{-4} cause some instability to appear in the solution. With moderate filtering however, the SNR required in the displacement data is within practical limits for MRI acquisition. Use of standard regularization methods to better manage this ill-conditioning will be discussed. The sensitivity to noise perturbation also suggests that it may not be possible to reconstruct sharp-edged features in the modulus distribution by this method.

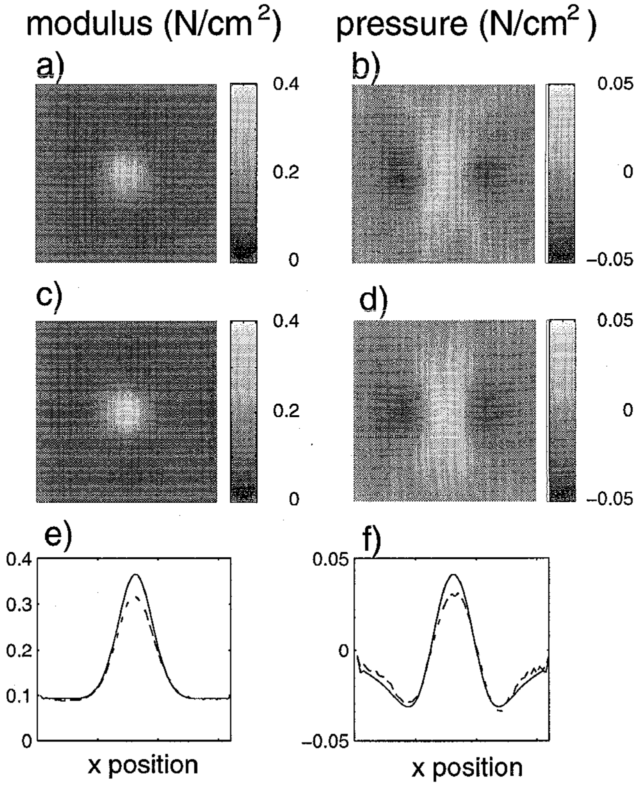

Figure 1. Results of the pressure/modulus inversion corresponding to unfiltered SNR=200 in the displacement data. Figures 1a-b show the modulus and pressure reconstructions, while figures c-d show the true modulus and pressure fields. Horizontal profile plots through the above images are shown in e-f for the calculated quantities (- - -) and true quantities (—).

References

[1] Sarvazyan A, Goukassian D, and Maevsky G. *et al. in* "Proceedings of International Workshop on Interaction of Ultrasound with Biological Media", pp. 69–81. Valenciennes, France, 5-9 April 1994.

[2] Chenevert T, Skovoroda A, O'Donnell M, and Emelianov S. *Magn. Reson. Med.* 1998; **39**:482–490.