Fast Optimization of a Biplanar Gradient Coil Set

D. Tomasi, E.C. Caparelli, and H. Panepucci†

Escuela de Ciencia y Tecnologia, Universidad Nacional de General San Martin, Alem 3901, Buenos Aires (1651), Argentina.

† *Instituto de Física de São CarlosUniversidade de São Paulo, Av. Dr. Carlos Botelho 1465CEP 13560-250- SP - Brasil*

The standard simulated annealing (SA) procedure, to optimize shielded gradient coils, uses n free parameters to model the primary coil and m extra free parameters for the shielding coil [1], which are randomly varied. At each annealing step the gradient homogeneity and coil efficiency are calculated inside the primary coil, and the shielding proformance is evaluated outside to the shielding coil to optimize the wire arrangement using the annealing procedure described above. In this work we are proposing a fast simulated annealing method where the shielding current distribution is analytically derived and simple geometries for wires are employed, allowing the use of analytic expresions to calculate the gradient field inside the primary coil. This procedure do not increase the necesary number of degree of freedom in the annealing problem and fastly converges to the local minimum of the error function E, resulting in a significant reduction of the computing time.

As we have shown in Ref. [2] a set of shielded biplanar gradient coils can be achieved considering a set of four planes perpendicular to the z–axis where the shielding planes and the primary planes are placed at $z = \pm d$ and $z = \pm a$ $(d > a)$, respectively.

The longitudinal gradient coil was modeled using n circular wires with radius R_i $i = 1 \ldots n$ placed in each primary plane. The wire distribution in the primary planes must be identical to maximize the field uniformity, but the currents on each plane must be opposite to produce an axial gradient.

Several authors [1] have obtained simulated annealing optimized shielded coils adding m extra degrees of freedom into the annealing algorithm, to null the magnetic field outside the shielding coil. This increases the computing time for each annealing–step and also the number of steps needed to reach the local minimum of E.

To reduce the CPU time necessary to achieve this local minimum we reduced the problem degree of freedom to n in the following way. To get the shielding coil we used the shielding density, $j^d(r)$, derived from the planar target field method, which for $n = 1$ is related to the primary current by [2]

$$j^d(r) = -IR \int_0^\infty \xi \frac{\sinh(a\xi)}{\sinh(d\xi)} J_1(R\xi) J_1(r\xi)\, d\xi, \qquad (1)$$

where r is the axial distance, $J_1(x)$ is the Bessel function of order 1, and R is the radius of the wire carrying a current I. This relation between currents flowing in the primary and the shielding planes can be used to obtain, by superposition, the current density to shield a coil consisting of a set of circular wires of radius $\{R_i\}$.

Another strategy followed to reduce the CPU time was to look for regions where the gradient fields could be analytically evaluated. Unfortunately this methodology not always can be used, but in this case the gradient field can be expresed analytically along the z–axis.

We wrote a C–code performing the annealing of a set of n wires on each primary plane.

The optimization procedure produces a significant reduction of the error function, which for the final configurations essentially does not depend on the initial set $\{R_i\}$. Furthermore, due to the probabilistic nature of the test used to accept or reject a given configuration, the algorithm can deal with relative minima without getting traped.

To model the y–transverse–gradient–coil we used $2n$ straight wires of length $4a$ at $\pm y_i$ $i = 1 \ldots n$ on each primary plane. In this case, the current must 0flow along x in the same direction to produce the desired gradient field.

Similarly to the axial case, to calculate the current density, $j^d_x(y)$, on the shielding coil we used the analytical relation between the primary and shielding densities, where for $n = 1$ can be written as [2],

$$j^d_x(y) = -\frac{2I}{\pi} \int_0^\infty \cos(y_1\xi)\cos(y\xi) \frac{\cosh(a\xi)}{\cosh(d\xi)}\, d\xi. \qquad (2)$$

The positions of the wires used to mimic this current distribution in the shielding coil, were calculated as in the longitudinal gradient case. As shown in [2] the shielding coil can be used as a return path for the primary coil current, therefore no extra wires must be included in the calculation.

Because of symmetry, the y–component of the gradient field produced by the k-wire have an analytical expresion on the $x = 0$ plane.

As for the longitudinal gradient case we developed a simulated annealing C–code to optimize the transverse gradient coil.

As before the error function evolves to a minimum and the final optimized configurations do not depend on the initial wire arrangement.

To study the effects of the number of turns on the coil performance, for the longitudinal and transverse cases, we varied the number of wires n from 1 to 40 and we evaluated the gradient field per unit current, η, the coil inductance, L, and the gradient field homogeneity.

The overall performance of coils was evaluated by a relative figure of merit which have shown an initial increase with n due to the fast improvement of the homogeneity and a decrease at large n which results from the increase of L. A maximum was observed in this figure of merit which indicates that an optimum n exist for a given FOV.

SUMMARY

The SA method can greately improve the design of high performance gradient coils. When applied to the optimization of shielding coils the increase in the necesary number of degree of freedom demands a very heavy computational task resulting in long computing times. As shown in this paper the use of the analytical relationship between primary and shielding currents allows a great reduction in the number of degree of freedom.

The computing time can be further reduced by restricting the wire geometries to those that allow analytical calculation of the magnetic field over selected regions that can then be used to evaluate the error function used in the annealing precedure. The examples presented in this paper show that this description does not impoverish the quality of the result.

The present approach resulted in approximately 30 minutes of computing time even using a non optimized C–code running in a standard 166MHz PENTIUM PC under DOS.

Additionaly we have derived generic expressions for the inductance of shielded biplanar gradient coils, which were applied to calculate the coil inductance and used to show that an optimum number of wires exist for this geometry.

[1] Crozier, S.; Doddrell, D.M. Gradient-coil design by simulated annealing. *J. Magn. Reson. A* 103, 354-357, 1993.

[2] Caparelli, E.C.; Tomasi, D.; Panepucci, H. Shielded biplanar gradient coil design. *J. Magn. Reson. Imaging* in press, 1998.