Introduction. The classic gradient coil of Golay [1] is simple to build and has good efficiency. However, these gradients are relatively long. Turner [2,3] and others introduced Fourier Space methods for rapidly designing minimum-inductance gradient coils. It is difficult but possible to introduce length constraints or asymmetric FOV into these fast Fourier techniques [4]. To more easily accommodate length, asymmetry and current constraints, others [5,6,7] have used "Coil Space" methods using nonlinear optimization, which tend to be much slower than Fourier methods. Here we introduce an extremely fast coil space algorithm for designing minimum-power gradient coils with arbitrary geometric constraints.

Design Technique. Figure 1 illustrates an arbitrary mesh of feasible current locations. The drawing shows a cylindrical former with eightfold symmetry, but the mesh can be arbitrary. Each of the nodes on the mesh has 4 entering currents and 4 exiting currents. The algebraic sum of the currents must be zero (the Kirchhoff's Current Law, or KCL, constraint) to ensure that currents circulate. Gradient field homogeneity is constrained at several target points on the surface of the FOV. The field can be expressed as $B_t = A_i$, where $A_{mn}$ is the field at the $m$th target point due to a unit amplitude current through the $n$th feasible current segment connecting two adjacent nodes. The power of a series wound gradient coil is

$$P = \sum_{n=1}^{N} |i_n|^2 \left( \frac{l_n}{\sigma_c S_n} \right) = \left( \frac{J}{\sigma_c} \right) \sum_{n=1}^{N} l_n |i_n|$$ (1)

where $J$ is the current density, $\sigma_c$ is the wire conductivity, $S_n$ and $l_n$ are the cross-section area and length of each current segment. Hence, we design a minimum power gradient set by solving the following optimization problem:

Minimize $\sum_{n=1}^{N} l_n |i_n|$  
Subject to $|Ai - B_d| \leq B_d \epsilon$, $Q_i = 0$,

where $Q$ is a "bookkeeping" matrix that enforces the KCL constraints, and $\epsilon$ is typically between 0.05 and 0.10, depending on the application. This problem is known as an "$\ell^1$-norm" problem, which can be solved efficiently using linear programming. The solutions automatically have the minimum number of loops necessary. If maximum current limits are imposed the windings become distributed. Using a linear programming package called PCx [8], we can design a transverse gradient coil with 300 nodes in less than 1 minute. Additional target points can also be included to design actively shielded gradient coils [9,10].

Results. Figure 1 also shows an 80-cm-diameter, 10 mT/m $G_x$ gradient designed with our algorithm. The mesh length was constrained to be 90 cm. The gradient is uniform to 5% at the target points on a sphere of diameter 32 cm, as shown in the contour plots in Fig. 2. This gradient is half as long as an equivalent Golay set. This length reduction comes at the cost of a 50% reduction in efficiency, measured in mT/m/A.

Summary. We have introduced a fast algorithm for designing minimum-power gradient coils that are simple to construct. The algorithm is flexible enough to allow arbitrary geometrical constraints on the coil former surface (e.g., constrained length, variable radius former). It can also handle asymmetric FOVs, arbitrarily shaped FOVs, and maximum current constraints.

Figure 1: Current mesh for gradient design (left). One octant of a constrained length 10 mT/m $x$ gradient coil (right).

Figure 2: Simulated contour plots of the $x$ gradient coil shown above, measured in Gauss. Left is at the plane $z = 0$; right is at the plane $y = 0$.

References