

Truncation Artifact Reduction in MRI with a New Class of Filters

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Introduction

It is well known that near the discontinuities in a cross-section, unfiltered MRI reconstruction show Gibbs artifacts [1], which are evident in most MR images on sufficient magnification [2]. These artifacts appears as series of ripples propagating away from the point of discontinuity with ripple spacing roughly twice the pixel size [3].

In this study, We introduce a new class of high order filters. A good filter should possess following properties : (1) reduction of truncation artifacts, (2) gives good contrast, (3) convergence of filtered image, (4) no reduction in S/N ratio and (5) simplicity of implementation. Our filters have shown to possess all these properties. The results are compared with conventionally used filters like Gaussian, Fermi and Hanning. As compared to conventionally used filters, our filters have shown remarkable reduction in truncation artifacts and an improvement of resolution.

Method

We represent the actual cross-section by $f(x,y)$, where $(x,y) \in [-\pi,\pi] \times [-\pi,\pi]$, frequency domain signal by $\hat{f}(m,n)$ $-\frac{N}{2} \leq n \leq \frac{N}{2}, -\frac{M}{2} \leq m \leq \frac{M}{2}$ and $M < N$. Since in MRI, we can take more data in frequency encoding direction without any substantial time loss, truncation artifacts are analyzed only in phase encoding direction. A filtered reconstruction is defined as $f_N^R(x,y)$ by the following relation :

$$f_{M,N}^R(x,y) = \sum_{n=-N/2}^{N/2} \sum_{m=-M/2}^{M/2} W\left(\frac{m}{M/2}\right) \hat{f}(m,n) e^{i(mx+ny)} \quad \dots (1)$$

where $W\left(\frac{m}{M/2}\right)$ is the window function.

We define the class of our windows by

$$W_{2l}\left(\frac{k}{M/2}\right) = {}^{2l}C_1 \cos\left(\frac{k\pi}{M+1}\right) {}^{2l}C_2 \cos^2\left(\frac{k\pi}{M+1}\right) + \dots + {}^{2l}C_{2l} \cos^{2l}\left(\frac{k\pi}{M+1}\right)$$

We call this family of filters as HBRR (Hanning Bernstein-Rogosinski RKS Rathore) class. For $l=k$, the filter is denoted by HBRR_k. These filters are modification of Hanning or Bernstein-Rogosinski operators and can be shown to admit following error estimates

Theorem 1 : For every 2π -periodic function $f \in L_p(I)$, $p > 1$, the relation

$$\|f_{n,2l}^* - f\|_{L_p(I)} \leq CE_n + \frac{1}{2^{2l}} \omega_{4l}\left(f, \frac{\pi}{2n+1}, p, l\right) \quad \dots (2)$$

holds true. Where $C = \left[\left(\sqrt{l+1}\right)\pi + 1\right] 2^{2l-1}$ and E_n being the best approximation to $f(x)$ by trigonometric polynomials of degree at most n (highest order frequency in data) and $\omega_{4l}(f, \delta, p, l)$ being the $4l^{\text{th}}$ order modulus of smoothness of f in $L_p(I)$ norm.

The above estimates ensure the convergence and high order approximation to smooth regions by HBRR class of filters. Although these filters are implemented in frequency domain, they can also be implemented with equal ease in spatial domain from an enlarged image obtained through zero padding the frequency data.

Results and Discussions

We have analysed the behaviour of filtered reconstruction of a sub-region of a MR image with different filters taking 96X256 Fourier data. The shape and PSF functions of the filters are shown in the Fig 2. and Fig 3 respectively. To eliminate Gibbs artifacts, a window should not be discontinuous at the boundaries [4]. But this is not the case with Fermi window (Fig 2), as it can be seen from the reconstruction also (Fig 1), Fermi filter does not reduce Gibbs artifacts satisfactorily. Although, stronger versions of Gaussian and Hanning windows

reduce Gibbs artifacts, they increase the blurring as compared to HBRR_1 filter. Experiments with HBRR class of windows have shown remarkable reduction in truncation artifacts along with better resolution as compared to other filters. Hence there is a two fold improvement in the quality of the reconstructed image with our class of filters. With

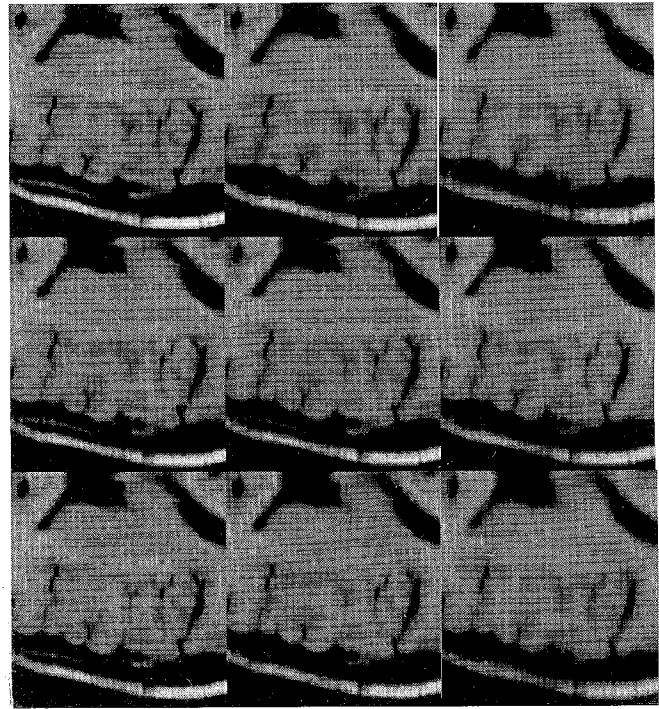


Fig 1. Filtered reconstruction with 96X256 Fourier data
Row 1 : Unfiltered , HBRR_1, Hanning
Row 2 : Fermi(Weak) ,Fermi(Medium), Fermi(Strong)
Row 3: Gaussian(W) ,Gaussian (Medium), Gaussian(Strong)

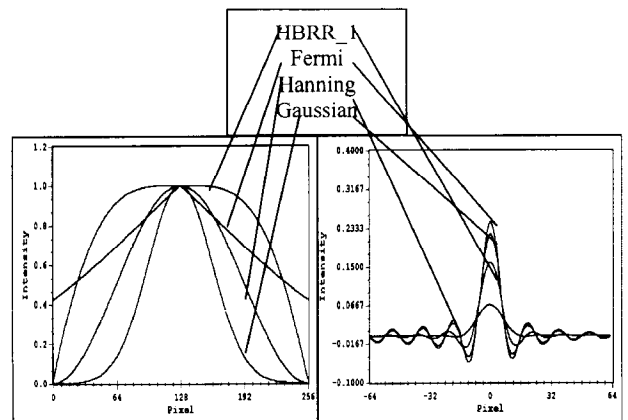


Fig 2. Shape of windows
Fig 3. PSF of filters
the help of experimental and theoretical justification (Theorem 1) we recommend the usefulness of HBRR class of filters for MRI images.

References

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