Signal-to-Noise Ratio and Signal-to-Noise Efficiency in SMASH Imaging

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Introduction

It is a well-established rule of thumb in magnetic resonance imaging that, all other things being equal, signal-to-noise ratio (SNR) tends to decrease as the image acquisition time is decreased. For the case of an imaging system operating at a fixed level of performance (e.g. at fixed gradient slew rates), and neglecting the effects of relaxation, this rule of thumb has been formalized into a rigorous principle: for a given voxel size, SNR in traditional MR imaging is proportional to the square root of the acquisition time. This proportionality has been shown to arise from the integration over acquisition time that effectively takes place during Fourier transformation of the MR signal in the course of image generation (1).

Recently, however, *in vivo* MR images have been obtained in substantially reduced acquisition times without appreciable sacrifices in SNR, using a partially parallel imaging strategy called SiMultaneous Acquisition of Spatial Harmonics (SMASH) (2-3). This would appear to violate the basic proportionality outlined above. It is the purpose of this work to clarify the preliminary results of (2-3), and to demonstrate that there is no violation, so long as the principles underlying the relation between SNR and acquisition time are suitably generalized to account for partially parallel MR acquisitions.

Methods

SMASH image reconstructions involve the coordinated use of multiple MR signals simultaneously recorded in the component coils of an RF coil array. Linear combinations of component coil signals are used to generate composite signals shifted by well-defined amounts in *k*-space, and these shifted signals take the place of data which would otherwise have required additional phase encoding gradient steps. The success of this procedure relies upon the presence of *signal correlations* among component coils in the array, such that component coil signals acquired at a given point in *k*-space (i.e. at a given encoding gradient strength) may in fact be related through appropriate combinations to signals at adjacent points in *k*-space. It is the presence of these signal correlations, *in the absence of concomitant noise correlations*, which allows for the preserved SNR which may be achieved using SMASH.

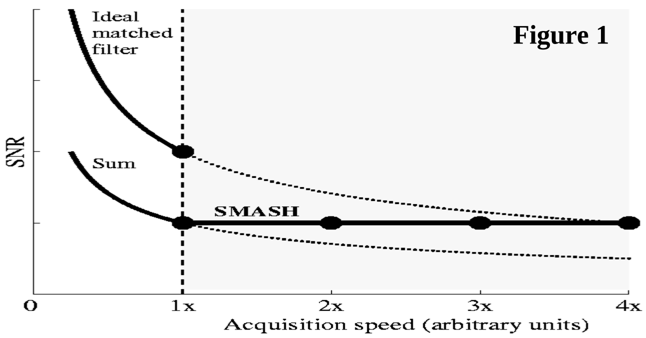
In order to study the behavior of noise correlations in SMASH image reconstructions, an analytic theory of noise propagation was developed, and the theory was tested in simulations and phantom imaging experiments. Effects of existing noise correlations among residually coupled array elements were included in this theory, along with the effects of any new correlations which might be introduced by the reconstruction procedure.

SNR in SMASH reconstructions of simulations and phantom images was then compared with the SNR of reference images obtained using traditional sequential imaging techniques. Since SNR in phased array imaging depends sensitively upon the algorithm used to combine component coil images, several different SNR references were used, reflecting several different combination strategies, including *i)* a simple weighted sum of component coils images, with a single weight for each coil, and *ii)* the optimal pixel-by-pixel combination described by Roemer et. al. in (4). The latter procedure involves weighting the image from each component coil by the complex conjugate of its RF sensitivity function prior to combination – a procedure which may be identified as a form of matched filtering.

Results

Our studies of noise propagation indicated that when the weights used for SMASH reconstruction of different *k*-space lines form an orthonormal set, no additional noise correlations are introduced by the reconstruction, and the noise variance is equal to that obtained from summation of the full reference data sets. When the SMASH weight sets depart from orthogonality, on the other hand, the noise contents of different *k*-space lines are partially correlated. Fourier transformation then leads to a combination of correlations *and anticorrelations*, which in turn produce oscillations of the noise variance around the reference value. In this case, some areas of the reconstructed image have anomalously high noise variance, whereas others have correspondingly low variance, preserving a constant mean. With appropriate coil array design, it is possible that this interesting property could be exploited to produce SNR “hotspots.”

Figure 1 summarizes schematically the results of SNR comparisons for the case of orthonormal SMASH weights in a four-element array. As expected, the reference SNR for simple summation of component coil images is seen to decrease as the square root of acquisition time. Reference images based on an idealized version of matched filtering begin at higher SNR, but also follow a square root decay with decreasing acquisition time. SMASH starts at the lower SNR of the simple sum, but maintains a constant SNR with increasing acquisition speed, so that at the maximum achievable acceleration factors it has crossed entirely from one curve to the other. Assuming that the reference images are obtained near the speed limit of traditional imaging (indicated by the vertical dashed line in the figure), this constitutes a significant and otherwise unattainable increase in imaging speed without a sacrifice in SNR efficiency (defined as the SNR per unit square root acquisition time).



Discussion and Conclusions

The SNR measurements reported in (2-3) actually showed comparable absolute SNR for SMASH and a reference image approximating the optimal matched filter. These measurements were correctly reported, but used an empirical SNR estimation technique which did not take into account the particulars of array combination. The analytic theory and the measurements presented here indicate that, when compared rigorously with pixel-by-pixel array combination techniques, SMASH reconstructions are expected to display a net SNR loss image-by-image, but, in optimized implementations, a preserved SNR efficiency. Thus, SMASH imaging offers many of the advantages of traditional phased array imaging, while allowing significant improvements in temporal resolution and corresponding reductions in artifacts due to physiologic motion and spin relaxation.

There is at least one case in which the effects of spin relaxation may in fact allow substantial *increases* in SNR efficiency using SMASH. In single-shot imaging sequences, the potential SNR gains associated with faster imaging and reduced T_2 or T_2* relaxation may often outweigh any losses associated with SMASH reconstruction. The use of SMASH with ultrafast single-shot sequences is a subject of ongoing study.

References

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