Reconstruction of MR Images from Data Sampled along
Arbitrary k-Space Trajectories

V. Rasche, R. Proksa
Philips Research, Division Technical Systems Hamburg, D-22335 Hamburg, Germany

Introduction

Recently MR acquisition techniques sampling data on non-rectilinear grids such as spiral [1] or projection reconstruction techniques [2] have become of major interest for MR fluoroscopy. For reconstruction of data sets obtained by these techniques convolution reconstruction techniques or gridding [3,4] are highly attractive for re-sampling of the non-rectilinear k-space data onto a rectilinear grid. During convolution a compensation for the for the non-uniform sampling density must be applied to avoid artifacts in the reconstructed image. For many trajectories the density compensation function (DCF) is not analytically known since the trajectory through k-space may be significantly corrupted [5]. A comprehensive overview of techniques for estimation of the DCF is given in [6].

We like to present an alternative approach for the estimation of the DCF using the area of the unity cell around each sample as a measure for the sampling density.

Methods

The determination of the unity cells for a given sampling distribution is done by calculating the Voronoi diagram which is defined according to Aurenhammer [7] by:

Let S denote a set of n points (sites) in the plane. For two distinct sites p, q in S, the dominance of p over q is defined as the subset of the plane being at least as close to p as to q. Formally:

\[ \text{dom}(p,q) = \{ x \in \mathbb{R}^2 | \delta(x,p) \leq \delta(x,q) \}, \]

for \( \delta \) denoting the euclidean distance function. Clearly \( \text{dom}(p,q) \) is a closed half plane bounded by the perpendicular bisector of p and q and will be termed the separator of p and q. The region of a site p in S is the portion of the plane lying in all of the dominances of p over the remaining sites in S. Formally

\[ \text{reg}(p) = \bigcap_{q \in S \setminus \{p\}} \text{dom}(p,q) \quad (1) \]

A partition of the plane according to (1) is called the Voronoi diagram.

For reconstruction the area of \( \text{reg}(p) \) was used as the reciprocal value of the DCF at sample p. The Voronoi diagram was calculated using a slightly modified version of the public domain software QHULL [8].

Results

We applied the introduced DCF for reconstruction of three data-sets acquired on trajectories as (a) projection reconstruction (PR) trajectory, (b) constant angular speed spiral (STI) trajectory, and (c) an arbitrary (AT) trajectory. In the latter case a two-fold over-sampling was applied to avoid artifacts caused by not fulfilling the Nyquist theorem.

Figure 1 summarizes the performance of the introduced technique. Figures 1b,d,f show the center region of the resulting Voronoi diagrams, and Figs. 1a,c,e the corresponding reconstructed images. For all cases the DCF yields very promising results. In the PR and the STI case the calculated DCF allows nearly perfect reconstruction of the simulated MR data set. The slightly remaining modulations in the AT case indicate

References