RF Field Transmission: $B_1$ Field Non-Uniformity, & SAR

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Highlights:

• Parallel transmit is now thought of as a constrained optimization problem in which the degrees of freedom present in the transmit array and pulse excitation are exploited to achieve a better excitation pattern and/or lowering local and/or global SAR. The excitation shape is optimized (to be as close as possible to the target pattern) given the constraints imposed by RF hardware (such as average and peak RF voltage) and externally imposed global and local SAR limits. Ideally, every constraint you care about should be included. If local SAR is constrained in every 10g region this leads to hundreds of thousands of constraints and is computationally burdensome.

• The quadratic form of the local SAR constraint allows us to compress these $\sim 10^4$ local SAR constraints down to a few hundred (computationally manageable constraints) and always be confident that we have conservatively estimated local SAR. I.e. the lossy nature of the compression will never lead to a higher than expected SAR.

• In such an optimization problem, success depends on having sufficient Degrees of Freedom (DoF) to generate a good optimum. Parallel transmit arrays offer an obvious source of spatial degrees of freedom since an independent waveform can be sent to each array element, but other DoF can be generated by varying the gradient trajectory used during excitation, and the exclusion of “don’t care” spatial areas from the optimization or allowing some spatial phase variation in the excitation.

• While Maxwell tells us that E and B fields come together as pairs, the E field is shaped much more strongly by the geometry of the conductive tissues in the body. This leads away from the thinking that the E field is a necessary, if unwanted byproduct of excitation. It can be confined and steered away from troublesome conductive geometries.

• While we have historically focused almost exclusively on $B_1^+$ since it is the only component that causes spin excitation, the other two components of $B_1$ ($B_1^-$ and $B_2$) also produce E fields (and thus SAR). These “Dark Modes” can be exploited to cancel electric fields associated with the primary excitation.

• With sufficient DoF in the transmit array, multiple excitation waveforms can be created which produce very similar excitation patterns but different local SAR patterns. Therefore the standard practice of using the same excitation pulse (except for a frequency offset) for every slice in an MR acquisition is sub-optimal. Better is to expand DoFs by allowing a different pulse waveform set for every slice’s excitation and jointly optimize the hundreds of waveforms to constrain 2 and 6 minute averaged local SAR; the more appropriate thermal constraint as recognized by the regulatory agencies. This “SAR hopping” optimization encourages the local SAR hotspot to “hop” around and temporally average to a lower value.
**Target Audience:** MR physicists interested in developing or using parallel transmit methods.

**Outcome/Objectives:** Update ideas of what is possible in RF excitation optimization and generate an enthusiasm for finding and exploiting DoF in the transmit process.

**Introduction:** Excitation of the MR spin systems through the application of a resonant radio frequency (RF) magnetic field started as a fairly straightforward part of MRI to conceptualize. The goal was to deliver a uniform excitation field, B₁, of the proper circular polarization (B₁⁺) to create spin excitation. Since the flip angle is the primary determinant of many contrasts and pulse-sequence effects in MRI and is linear in B₁⁺ for small tip excitations, choosing a uniform excitation field and thus uniform flip angle distribution across the body is a desirable goal. This meant choosing a resonant structure that inherently generated a uniform excitation field. For the long wavelength limit (λ < body dimensions), a birdcage like structure with a sinusoidal distribution of current around the rungs efficiently achieves this goal. SAR was considered a necessary evil that accompanied successful excitation. Although pulse sequences and RF pulses could be optimized to lower SAR, the transmit system was relatively inflexible in its ability to produce excitation but not SAR since Maxwell’s equations state that the presence of an oscillating magnetic field (B₁) must cause an electric field (E₁) which causes the currents in the conductive tissue leading to SAR and its undesirable heating effects. Since the spatial distribution of B₁ is determined by our uniform excitation needs, then there is not much we can do about SAR short of making sure that stray electric fields near capacitors did not interact with the body.

The superficial simplicity of this picture began to unravel with the advent of ultra-high field MRI. Wave propagation effects were quickly identified as the source of the excitation inhomogenieties. [1, 2] The Figure to the right shows a propagation effect in a birdcage coil. As the wave propagates inward from the rungs, it acquires appreciable phase as it travels a significant fraction of its wavelength in the 7T situation, but less so at 3T. This leads to the swirling pattern in the B₁+ phase map. Like many “problems” a new effect (wave propagation) also leads to benefits, such as improved g-factors at higher fields [3] and novel traveling wave excitation methods.[4, 5]
Methods: In conventional pTx pulse design, the 2D or 3D excitation pattern created by a pulse is expressed as a vector which is computed from a forward linear model; the discretized integral equation describing the RF excitation in the low-flip regime. [6] Thus the magnetization excitation pattern vector, $m$, is the product of an encoding matrix, $A$, describing the modulations from the gradient and B1+ profiles and the RF waveform vector; the complex RF amplitudes played out at each time-point as depicted below. The waveform vectors for each array element are stacked. The linear equations are solved by a rapid minimization of a cost function that is the sum of a least-squares (LS) error term ($2$-norm of the difference between the target and achieved spatial excitation patterns (excitation fidelity) and a SAR or RF power quantity (penalty term).

The trade-off between minimization of the LS error and the penalty term is controlled by the regularization strength, or Lagrange multiplier. One way to optimize the Lagrange multiplier is to step through a range of values and plotting the excitation fidelity as a function of the SAR or power quantity penalized (L-curves). Such a procedure is cumbersome however, especially when multiple quantities are penalized. On the other hand, sub-optimal choice of the regularization parameters results in sub-optimal excitation fidelity or excessive SAR or power, or both.

A more natural approach is to simply ask for the best possible excitation fidelity subject to a list of hard constraints.[7] I.e. to ask the optimizer to “give me the best pulse consistent with SAR and amplifier hardware limits (list of explicit constraints).” Note this is subtly different from the regularization approach which effectively asks: “give me the pulse which minimizes the sum of a fidelity error term, two SAR terms and two RF power terms weighted by numbers which I guessed at (the regularization parameters.)

In the constrained optimization approach, the relevant constraints include the global SAR, which is expressed from the global SAR matrix $Q$ which is calculated from the E fields for each coil and the tissue conductivity map as: $x^*Qx$. [8, 9] Each 10g parcel of tissue can similarly get a Q matrix and be used as an independent constraint (i.e. they must all be kept below the maximum local SAR limit.) In practice, the full list of local SAR constraints is too big (~$10^4$ constraints) to be practically manageable. But the local SAR matrices can be compressed to a few hundred constraints.[10, 11] Importantly, the compression is done in a way that guarantees that the SAR constraint is met. Namely any errors result in a conservative, over-estimate of SAR rather than a dangerous under-estimation. [11] Finally, RF peak and average power constraints imposed by the RF amplifiers can easily be computed as the peak and average of the 2-norm of $x$.

In the “Dark Mode” approach we show that array elements that produce no meaningful spin excitation (have low $B_1^+$ efficiency) can none-the-less be useful to excitation. Namely, these “dark modes can be energized to try to cancel a local SAR hot-spot produced by the array elements that do perform excitation. Although the optimization can be phrased as “first create your best excitation with the “bright modes” and then try to reduce local SAR by energizing the
“dark modes”, in fact, the full optimization strategy will find the best solution without distinguishing “bright” and “dark”. Nonetheless, it expands our thinking about arrays to consider that the elements might have some role beyond B1+ field production.

**Results:** Results will be shown that demonstrate the power of the constrained optimization when armed with sufficient DoFs. Firstly we learn that it is important to constrain what you care about, such as global and local SAR, and not proxies for them such as peak and average RF power. Secondly we learn the power of creating new DoFs such as relaxing constraints where we don’t care, such as eliminating the phase from the target excitation pattern (MLS instead of LS optimization). Also, adding degrees of freedom to the gradient waveform used during the pulse is also very powerful. It is important to remember that pTx does much more than just sculpt the excitation field (B1 shimming does this). Parallel transmit sculpts the excitation magnetization, phase and amplitude, and to do this it relies heavily on the phase modulation produced by the gradient trajectory. Taken together, pTx excitations can produce both better excitation flatness and lower SAR (both global and local) than a traditional birdcage body coil.

**References:**

8. Zhu, Y. **RF power deposition and "g-factor" in Parallel Transmit.** in Proceedings of the ISMRM. 2006. 599
