Fourier Domain Approximation for Bloch Siegert Shift

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Target audience: Researchers using the B1 mapping method based on Bloch-Siegert shift.

Purpose: In this study, we propose a new simple Fourier domain based analytical expression for the Bloch-Siegert (BS) phase shift based B1 mapping method. In this expression, the phase is calculated in terms of Fourier transform of the RF pulse envelope and therefore off- and on-resonance effects can be understood more easily. It is shown that $|B1|$ can be obtained more accurately by the aid of this expression while employing short BS pulse durations and small off-resonance frequencies.

Theory: In the BS shift based B1 mapping method, off-resonant RF pulse is applied after the excitation RF pulse in order to add phase shift to the excited spins. The amount of the phase shift depends on the envelope of the RF pulse ($B1(t)$) and the frequency offset of the RF pulse ($\omega_{off}(t)$) from the resonance frequency ($\omega_0$). In [1], it is shown that when $\omega_{off}(t)$ is much higher than $|\omega_0(t)| = y|B1(t)|$ where $y$ is the gyromagnetic ratio, then in the $\omega_0$ rotating frame, the phase shift is directly related to the time integral of the square of $|\omega_0(t)|$ and inversely related to the offset frequency as given in Eq. (1).

Since long BS pulse durations cause long phase values and longer offset values result in signal loss due to the $T1^*$ effects, use of small pulse duration becomes important. In an earlier study [2], it was shown that when small pulse duration is used, there is a significant difference between the actual phase shift ($\phi_{actual}$) and the time-domain approximation given by Eq. (1). This residue ($\phi_{res}$) can be calculated if the Bloch equations are solved in the $\omega_0+\omega(t)$ rotating frame, since by doing so the phase accumulating due to $\omega(t)$ is excluded from the actual phase shift in the $\omega_0$ rotating frame. In order to simplify the solution of the Bloch equations, a new magnetization vector which includes the phase changes under $\omega_f(t)$ and the magnitude $M_0$ rotating frame is defined with the initial condition $(M_0(t_0), M_0(t_0), 0, 0) = (M_0, 0, 0)$.

$$\phi_{res} = \int_0^T \frac{\omega(t)B1(t)}{2\omega_{off}(t)}dt \approx \int_0^T \omega_B(t)\frac{B1(t)}{2\omega_{off}(t)}dt \quad (1)$$

In this condition the time derivative of $M_1$ is very small, and it is assumed that $M_1$ remains almost constant throughout the Bloch-Siegert RF pulse. Therefore, the system of differential equations is reduced to Eq. (3) where $\omega_0(t)$ and $\omega(t)$ are the real and imaginary parts of $\omega_0(t)$, respectively. Using $\omega(t)$ approximation the solution for the $M_1$ component for the pulse duration $T$ can be written as in Eq. (4). Since we assume that $M_1 = M_0$, and $M_1$ is small, $\phi_{actual}$ becomes approximately equal to $\phi_{res}$ when $\phi$ is defined in left-hand direction. In order to find the phase shift defined in the $\omega_0$ rotating frame ($\phi_{actual}$), we add $\int_0^{\omega(t)dt}$ term to $\phi_{res}$ as given in Eq. (2). However, due to $\omega_{off}(t) >> |\omega_0(t)|$ approximation, the final phase expression given in Eq. (5) is again the approximated solution for $\phi_{actual}$ and defined as the frequency domain BS approximated phase shift $\phi_{FD}$. In this expression $\Omega(t)$ is the Fourier transform of $\omega_0(t)$ and since the Hilbert transform of a function $g(t)$ at $t = 0$ is given as $-i/\pi g'(t)dt$, the final form of the expression is defined in terms of the Hilbert transform. In order to find the peak of $\phi_{FD}$ from the phase in $\omega_{off}(t) >> |\omega_0(t)|$ region Eq. (5) is changed to Eq. 6 where $\Omega(t) = \Omega_{Peak}(t)$.

$$\phi_{FD} = \int_0^T \omega_B(t)\frac{B1(t)}{2\omega_{off}(t)}dt \approx \int_0^T \Omega(t)B1(t)\frac{B1(t)}{2\omega_{off}(t)}dt \quad (2)$$

Experiments & Results: In order to verify the frequency domain B1 approximation (Eq. 5), Bloch simulations and MR experiments are performed for Hard, Fermi and Shinner-Le Roux (SLR) pulse shapes with different pulse durations. SLR pulse is designed with 0.5% passband ripple, 1% reject ripple, and 0.3 kHz bandwidth by using VESPA-RF pulse tool [3]. All experiments were performed using a 3 tesla Siemens 1Tm Trio scanner with a Siemens phantom. The imaging parameters were: slice thickness=5mm, FOV=200mm, TR=100ms and resolution=256x256. The body coil was used for RF transmission and a 12-channel Siemens head coil was used for the reception. For hard and Fermi pulse shapes, the pulse duration was varied between 150us and 2ms and for SLR pulse shape, the duration was varied between 300us and 2ms. For each pulse shape the offset frequency is fixed at 4 kHz. In Figure 1, we present a comparison of the phase shifts obtained through Bloch simulations, observed in the experiments, obtained by Eq. 1, and obtained by Eq. 5 for different pulse durations. As seen in the figure, the results of the experiments follow the results of the Bloch simulations as expected and the difference between the results of

To analyze the relation between the phase and $|B1|$ at different offset frequencies, experiments were performed by using hard pulse with offset frequencies of 50 Hz, 100 Hz, 1 kHz and 4 kHz with $|B1|$ values in $\omega_{off}(t) >> |\omega_0(t)|$ region. The phase shift values that were obtained using the frequency domain Eq. (5) and the time domain (Eq. 1) approximations were compared with the results of simulations and experiments (see Figure 2). Although at 1 and 4 kHz frequencies, all results match very closely, when the offset frequency is 100Hz, the results of Eq. 1 start to deviate from the results of Bloch equations and the experimental results whereas Eq. 5 gives similar results with the experiments. Table 1 shows the error analyses done by using frequency domain approximation (Eq. 5) and the simulations. These error analyses also verify that the frequency domain approximated relation gives better solution for $|B1|$ than the solution of the time domain approximated relation for lower offset frequencies. Table 1 shows that at low B1 frequencies, precise knowledge of the B1 field and therefore the frequency offset is critically important. In addition, use of crusher gradients is very important to reduce image artifacts.

Conclusion: In this study, a new simple frequency domain analytical expression is proposed for the BS shift. Using this expression, $|B1|$ values can be predicted from the phase data by using the frequency spectrum of the RF pulse. The method works well even for short pulse durations and offset frequencies.


Figure 1: Phase difference for different pulse durations for Hard pulse, Fermi pulse, and SLR pulse with 4kHz offset frequency.

Table 1: Error Analysis of the solutions of $\phi_{actual}$ & $\phi_{FD}$ relations with the results of simulations & experiments