Introduction: Parallel imaging is a robust acceleration technique based on extracting spatial information from subsampled and noisy multi-channel observations. Interpolation approaches such as GRAPPA [1] use correlation across channels and between adjacent k-space blocks to estimate finite impulse response (FIR) interpolation filters from central k-space (ACS) lines. However, they exhibit a noise amplification effect [2,3] that may be mitigated via regularization techniques including total-variation, ℓ_1 and ℓ_2 norms [3-5]. However, regularization can be computationally expensive and may not be effective in eliminating both the noise and ghost artifacts without blurring the images. Here, we propose a pre-processing technique based on structured low-rank matrix approximation via truncated singular value decomposition (TSVD), which is able to suppress noise and ghost artifacts efficiently. TSVD method has been previously used in parallel MRI to improve the conditioning of the system matrix [6] and to reconstruct k-space via matrix completion [7]. In contrast to previous work, here rank properties are used to denoise acquired data in a computationally simple pre-processing for GRAPPA reconstruction.

Theory: The calibration and reconstruction equations in GRAPPA can be written as \( y_{\text{acs}} = A_{\text{acs}} g \) and \( y_r = A_r g \) respectively, where \( A_{\text{acs}} \) and \( A_r \) are calibration and reconstruction matrices in Toeplitz-block-Toeplitz (TBT) form, and \( g \) represents interpolation filters. Since the coil sensitivities are smooth, they can be well-approximated by FIR filters of size \((m_h,n_g)\) in 2D k-space. It can be proven that when \( K > R + 2 \), there generically exist interpolation filters if \( A_{\text{acs}} \) (or \( A_r \)) has non-zero nullity equal to

\[
Km_h n_g - (m_h + R m_g - R)(n_h + n_g - 1)
\]  

where, \( K, R \) and \((m_h,n_g)\) represent the number of coils, uniform down-sampling factor applied only along \( m \) direction, and size of the interpolation filters, respectively. In addition, from ACS data we can create a matrix \( A_c \) that is a convolution operator formed from the ACS lines without down-sampling; the nullity of the Toeplitz matrix \( A_c \) is equal to \( R = 1 \). As a result, matrices \( A_{\text{acs}}, A_r \) and \( A_c \) have low rank in the noise-free case. In practice, noise distorts the low-rank property of these matrices; however, they can be well-approximated by the closest low-rank matrix via singular value thresholding.

Methods: In order to denoise the data, we used Cadzow’s algorithm [8], which applies TSVD and TBT averaging iteratively, on \( A_c \) and on \( A_r \) sequentially (Step 1 and Step 2). From (1), \( A_c \) has lower rank than \( A_r \) and can be better approximated via TSVD; therefore, the shared samples are used in \( A_r \) for in Step 2 (see Fig. 1). As a result, both the ACS data and the uniformly subsampled data are jointly denoised. Then, the GRAPPA method is used to interpolate the denoised signal.

Results: Twelve coil-pMRI data were simulated from Biot-Savart law coil sensitivities (matrix size: 256x256); 4-fold downsampling with 18 central k-space lines (ACS) were acquired for an effective subsampling ratio of 3.12. As shown in Fig. 2, the GRAPPA reconstruction (kernel size 4x5x12) exhibits severe noise and ghost artifacts. The thresholds for TSVD were chosen using Bartlett’s test [9]. Five iterations were sufficient. Fig. 2 shows that TSVD GRAPPA reconstructs the image with fewer artifacts than both GRAPPA and TV regularized GRAPPA [4] (here implemented using [10]); peak signal to noise ratios (PSNR) are given in Fig. 2.

Conclusions: We presented a pre-processing method based on low-rank approximation of structured matrices. The simple denoising step, used in conjunction with GRAPPA, reduces both noise and ghost artifacts without blurring.