Theory

Recently, we proposed the exploitation of MRI-inherent spatiotemporal symmetries for computation of small tip-angle pulses [1] in parallel transmission. The Bloch equation, neglecting the dissipative effects of T1 and T2*, which is an autonomous ordinary differential equation, and the reciprocity principle [4], the consequence of Faraday’s law for MRI, display spatiotemporal symmetries that allows one to consider signal acquisition and small tip angle excitation [5] along the very same trajectory as a Fourier pair in parallel MRI (Eq.1). The operation that transforms a shape \( \sigma \) into signals \( s(t) \), is the signal acquisition or the Fourier analysis and the inverse operation is the Fourier synthesis or small tip angle excitation. While the tip-angle pulses [1] in parallel transmission. The Bloch equation, neglecting the dissipative effects of T1 and T2*, which is an autonomous ordinary differential equation, and the reciprocity principle [4], the consequence of Faraday’s law for MRI, display spatiotemporal symmetries that allows one to consider signal acquisition and small tip angle excitation [5] along the very same trajectory as a Fourier pair in parallel MRI (Eq.1). The operation that transforms a shape \( \sigma \) into signals \( s(t) \), is the signal acquisition or the Fourier analysis and the inverse operation is the Fourier synthesis or small tip angle excitation. While the tip-angle pulses [1] in parallel transmission.

Methods

To use this theory for image reconstruction is the next logical step. A measured MRI signal dataset is used in a hypothetical excitation simulation designed to restore the reciprocity yielding the excited pattern as the reconstructed image. The algorithm is particularly well suited for parallel imaging as, within constraints of the small tip-angle approximation, the local RF field corresponds to a weighted sum of pulses from individual transmitters. This linearity constraint can again be easily imposed on a simulation. According to [6] for non-Cartesian Nyquist acceleration the Hermitian is needed as well, which conveniently is the simulation of the signal acquisition part of the very experiment used to obtain the data at hand.

Computational parallelisation can be achieved for forward and inverse operators withNr x Nt and Nr, correspondingly; where, Nc, Nr and Nt denote the number of channels, image matrix sites and k-space sample points, respectively. Additionally, it should be noted that consideration of \( b_0 \) and \( b_1 \) do not increase the complexity of the algorithm, as the kronerck product of the sensitivity matrix with the encoding matrix is never actually performed. Note further, that the simulator only covers the acquisition and excitation modules and the flexibility and completeness of simulators like jemris [7] is not required. A simple MRI “simulator” using rotation matrices computed in the Cayley-Klein manner [8] was programmed for CPUs with MPI and OpenMP parallelisation for C++ and MATLAB. Additionally, an MPI/OpenCL version was programmed for production on GPGPUs for real-time applications.

Results

Figure 1 shows the runtimes and memory consumption for different parallelisation schemes and data sets to illustrate the favourable scaling with respect to these variables. Figure 2 shows a typical “money-shot” reconstructed using the XXX method.

Discussion and Conclusion

The MRI machine and sequence traditionally Fourier transform a weighted magnetisation density function as an analogue computer while the appropriate image reconstruction applies the inverse transform. In contrast to reality, where the reciprocity principle is violated at any Larmor frequency, a perfect world can be designed in simulation for reversing MRI signal formation process for image reconstruction. The proposed method is conceptually simple and opens up a range of potentially very useful instruments. One of the many applications would be to reconstruct parts of the image by restricting the spin population used for simulation to a region of interest. Also one can reconstruct images in a progressing fashion by simulating more and more dense grids until convergence is reached thus optimising the reconstruction to the resolution capabilities of the k-space sampling trajectory and to real-time needs. This study has demonstrated that algorithmic inversion of the MRI signal formation process shows favourable performance for image reconstruction particularly in the case of non-Cartesian imaging in the presence of off-resonances, but more strikingly exhibits an orders of magnitude smaller memory footprint in parallel operation.

References