Cross-Calibration Accuracy Requirements for Prospective Motion Correction
Murat Aksoy1, Melyn Ooi1, Julian Maclaren1, and Roland Bammer2
1Center for Quantitative Neuroimaging, Department of Radiology, Stanford University, Stanford, CA, United States

Purpose: Cross-calibration of the optical tracking system and the scanner is an essential step in prospective motion correction [1]. Errors in cross-calibration cause inaccuracies in motion correction and subsequent motion artifacts in the image. However, cross-calibration can be time consuming. Thus, in a clinical setting, it is desirable to skip cross-calibration and use pre-calibrated rigid mounts to position the camera. However, rigid mounts and scanner specs can drift over time, invalidating the pre-calibration. Additionally, coil-mounted camera systems [2,3] rely on the availability of the accurate position of the patient table, which may not always be satisfied. For evaluating the robustness of these rigid-mount systems, it is desirable to establish standards on the required accuracy of cross-calibration. In this study, we assessed the accuracy requirements of cross-calibration using a simulation environment.

Methods: The computer simulation environment we used for this study is shown in (Fig. 1). Following the setups in [3] and [4], the camera (Fig 1a) was placed either on the coil (d=20cm) or on the scanner bore (d=40cm), and the marker (Fig 1d) was assumed to be on the head 10 cm away from the center of the head. The head was placed at the center of the scanner and was assumed to be a sphere with a 10cm radius (Fig 1b). In order to simulate cross-calibration errors in 3D, the location of the camera was perturbed incrementally with a maximum of 5mm translation or 5° rotation for three translational and three rotational components. Shaking and nodding motions were simulated such that the tip of the head followed circles of varying radii with the back of the head being the pivot point, as shown in Fig. 1c. For each simulated motion of the head, the motion update of the scanner was calculated using the ground truth and the perturbed cross-calibration information using the following formula [3]:

$$T_{x,y,z} = T_{x,y,z}^0 T_{x,y,z}^{true} T_{x,y,z}^{perturbed}$$

Here, $T_{x,y,z}^0$ is the scanner update, $T_{x,y,z}^{true}$ is the ground-truth or the perturbed cross-calibration matrix, $T_{x,y,z}^{perturbed}$ is the motion of the marker and $T_{x,y,z}^{true}$ is the position of the camera with respect to the marker. A perfectly accurate cross-calibration will yield no net displacement of the head with respect to the scanner after prospective correction of the scan volume. However, the perturbation of the cross-calibration matrix $T_{x,y,z}^{perturbed}$ will result in an inaccurate scan volume update $T_{x,y,z}^{perturbed}$ which will result in an error of the pixel positions of the head even after motion correction. Thus, to quantify the accuracy of motion correction, the maximum pixel position error over all pixels within the spherical head volume was calculated. Mathematically, the expression for the maximum pixel position error is:

$$\epsilon(\psi) = \max_{x,y,z \text{ head volume}} \max_{x,y,z} \left| T_{x,y,z}^{perturbed}(\phi, \psi) (T_{x,y,z}^{true}(\phi, \psi))^{-1} \right|$$

In this study, considering routinely used image resolutions, the maximum pixel position error was set to $\epsilon(\psi) = 0.5mm$. Head motion was simulated for $0°<\phi<20°$, $0°<\psi<360°$ (Fig 1c). For each head motion, the maximum allowable perturbation (i.e. error) in cross-calibration $T_{x,y,z}^{perturbed}$ was calculated for three translational and three rotational components separately that would give a maximum pixel position error $\epsilon(\psi) = 0.5mm$.

Analytical solutions: In this case, the simulations were simplified to 2D such that only z-rotation of the head was simulated, and the cross-calibration was perturbed in two translational and one rotational axis. This way, the dependence of the maximum pixel position error $\epsilon$ on the error in cross-calibration was determined analytically using equations [1] and [2].

Results: The results of numerical simulations are shown in Fig. 2. The graphs show that for a bore-mount setup (d=40cm), the cross-calibration has to be accurate within $0.3°(\Delta \phi < 0.3°)$ and $3\text{mm}(\Delta \psi < 3\text{mm})$ to correct for $\psi=10°$ head motion, and it has to be within $0.2°(\Delta \phi < 0.2°)$ and $1.4\text{mm}(\Delta \psi < 1.4\text{mm})$ to correct for $\psi=20°$ head motion. For the coil mount setup (d=20cm), the cross-calibration has to be accurate within $0.5°(\Delta \phi < 0.5°)$ and $3\text{mm}(\Delta \psi < 3\text{mm})$ to correct for $\psi=10°$ head motion, and it has to be within $0.3°(\Delta \phi < 0.3°)$ and $1.4\text{mm}(\Delta \psi < 1.4\text{mm})$ to correct for $\psi=20°$ head motion.

For the simplified 2D case, the dependence of the maximum pixel position error $\epsilon$ on the error in cross-calibration was determined to be:

$$\epsilon(\psi) = 1200 \sin \left( \frac{\psi}{2} \right) \sin \left( \frac{\phi}{2} \right), \epsilon(\psi) = 2A_x \sin \left( \frac{\psi}{2} \right), \epsilon(\psi) = 2A_y \sin \left( \frac{\psi}{2} \right)$$

Discussion & Conclusions: In this study, we provided the minimum cross-calibration requirements for prospective optical head motion correction. For the 2D analytical simulations, the analytical expressions (Eq. [3]) revealed no dependence on the baseline position of the camera $d$ (Fig. 1), and the perturbation of the cross-calibration on reality, the sensitivity of the cross-calibration will also depend on the orientation of the camera with respect to the scanner, the motion axes, and the position of the head. However, the reference numbers established in this study will still help in system design. One important implication of the results in this study is that if the scanner table location is used to re-calibrate a coil-mount optical tracking system, the accuracy of the table position reported by the scanner has to be within $1.4\text{mm}$ to correct for $20°$ head motion.