Using Extended Phase Graphs: Review and Examples
Brian A Hargreaves and Karla L Miller
1Radiology, Stanford University, Stanford, CA, United States, 2FMRIB Centre, Oxford University, Oxford, United Kingdom

**Purpose:** The extended-phase-graph (EPG) formalism [1-3], which allows efficient signal simulation for numerous commonly used pulse sequences, is summarized intuitively, with applications and software.

**Basics:** EPG decomposes magnetization within a voxel into a basis of transverse ($F_n$) and longitudinal ($Z_n$) states, as shown in Fig. 1, where $n$ is the number of “twists” or cycles across a voxel. Assuming all gradients dephase an integer number of cycles across a voxel and that spins are homogeneously distributed in the voxel, this basis is perfectly invertible. Because $M_x$ is complex and $M_z$ is real, $F_n$ is defined for positive and negative $n$, while $Z_n$ is defined for $n \geq 0$.

**Sequence Propagation:** During each TR, precession due to gradients or off-resonance causes the transverse magnetization to evolve to the next dephased state (Fig. 2), while magnetization in longitudinal states remains there until it is transferred to a transverse state by an RF pulse. Relaxation causes decay of transverse and longitudinal states, and recovery of the $Z_0$ state. RF pulses transfer magnetization between states of order $n$ ($F_n$, $F_0$ and $Z_0$). Diffusion attenuates states in proportion to $D \times n^2$, as well as the square of the actual gradient area. Using simple building blocks, complicated sequences can be built up easily, and at any time, the $F_0$ state corresponds to the observed signal.

**Examples:** Any sequence with integer dephasing across a voxel over a repetition can be simulated easily with EPG. Example of gradient-echo sequences are gradient-spoiled (FISP, GRE) sequences [4], RF-and-gradient spoiled (FLASH, SPGR, T1-FFE), reverse gradient-spoiled (PSIF, CE-FAST), and stimulated echo (STEAM). Spin-echo sequences with CPMG, non-CPMG, reduced and varying refocusing flip angles can all very easily be built up [5]. Diffusion effects can be simulated in all cases very easily, using the formulation provided by Wiegel et al. [6]. Matlab code and examples are at http://bmr.stanford.edu/epg.

**Discussion Points:** The EPG formalism allows efficient simulation of groups of spins within a voxel by transforming magnetization into a Fourier basis, which is very compact when gradients dephase an integer number of cycles across a voxel.

- Magnetization can easily be Fourier transformed between the EPG [F_n, Z_n] basis and a spatial spin distribution $M(r)$, allowing modeling in two domains.
- Static dephasing effects can be modeled with EPG; then transforming to $M(r)$. Each position in $M(r)$ corresponds to a specific amount of phase accrual per unit time.
- Higher states can be eliminated from simulation when their content approaches zero.
- Assuming N non-zero states, the EPG propagations from relaxation, RF nutation, gradients and diffusion can be expressed as a linear system, so that complicated steady states can be quickly calculated numerically.
- Motion can be modeled with constant (bulk) phase shifts at gradient steps.
- EPG can be extended to model higher-dimensional gradient dephasing.

**Summary:** The EPG formulation offers a powerful and simple means to simulate numerous pulse sequences and effects in MRI. The reader is encouraged to explore the tools and examples to gain understanding of this powerful technique.


---

**Figure 1:** EPG basis for $M_x$ and $M_z$. $F_n$ states consist of twists of $n$ cycles across a voxel, in opposite directions for positive and negative $n$. $Z_n$ states consist of sinusoidal variation of $M_z$ ($n=0$). Equations show simple Fourier transform relationship between EPG states and magnetization.

**Figure 2:** Alternate illustrations of magnetization propagation. (top) Coherence pathways showing when magnetization reaches different $F$ and $Z$ states during a sequence of RF and gradient pulses. (bottom) 3xN Matrix representation of $F$ and $Z$ states with transitions due to RF pulses (vertical mixing of $F$ and $Z$ states), gradients (shifts in $F$ states) and relaxation (decay of all states, plus recovery in $Z_0$ only). Gradients of opposite sign would shift $F$ states in the other direction.

**Figure 3:** Example simulations using EPG Matlab code. Each requires a few simple lines of code, and easily completes in a fraction of a second, where Bloch simulations may be more challenging.