Automatic $L_1$-SPIRiT Regularization Parameter Selection Using Monte-Carlo SURE

Daniel S Weller, Sathish Ramani, Jon-Fredrik Nielsen, and Jeffrey A Fessler

EECS, University of Michigan, Ann Arbor, MI, United States, BME, University of Michigan, Ann Arbor, MI, United States

Introduction

$L_1$-SPIRiT [1] incorporates $\ell_1$-norm regularization into the auto-calibrating parallel imaging reconstruction method SPIRiT [2]. Although such regularized methods promise to improve image quality, allowing greater undersampling, selecting an appropriate value for the regularization parameter can impede practical use. This paper proposes a parameter selection criterion based on Stein’s unbiased risk estimate (SURE) [3], which estimates the mean-squared error (MSE) without knowledge of the true signal. Traditionally, SURE may be difficult to compute for iterative nonlinear reconstruction algorithms. Recently, a SURE-guided iterative reconstruction [4] was developed for single-channel MRI; we extend this work to multi-channel L1-SPIRiT reconstruction with arbitrary k-space undersampling using a complex-valued Monte-Carlo-based approach. Here, we apply it to reconstruct real parallel-imaging data with varying levels of noise.

Theory

The $L_1$-SPIRiT estimator $\hat{x}(\lambda)$ minimizes the wavelet-domain joint sparsity of the multi-channel images from full k-space $x$ that preserves the acquired data $y$ (where the observations have noise covariance $\Omega$) and is consistent with the SPIRiT kernel convolution matrix $G$; i.e. $x = Gx$. Joint sparsity is enforced using wavelet-domain soft-thresholding with parameter $\lambda$; the choice of this parameter determines the sparsity (and the quality) of the reconstruction. We would like to choose $\lambda$ to minimize the MSE, but since only the acquired data is available, we use SPIRiT to provide the full k-space from the true values at the sample locations. Since the true values at the sample locations are unknown, we turn to SURE to approximate the MSE and form a Monte-Carlo estimate [5] using only a single realization of a complex iid zero-mean unit-variance random vector $h$ and two evaluations of $f_\epsilon(\lambda)$ and $f_\epsilon(\lambda+\epsilon b)$, where we use $\epsilon = 10^{-5}$.

Methods

We initially validated this approach using simulated data; here, we show our results for real data. We acquired T1-weighted brain data of a consenting subject using 3D SPGR (TR/TE/FA = 25ms/5.2ms/25°) on a 3 T GE scanner with 256x144 axial slice with 1.0×1.25 mm spatial resolution, and we Poisson-disc undersample the data by a factor of 4, retaining a 24x24 block of calibration data. The sparsifying transform for L1-SPIRiT is a four-level `db4' wavelet; each L1-SPIRiT reconstruction consists of 20 iterations of the POCS implementation [1]. Un-regularized SPIRiT uses 20 CG iterations. The SURE values depicted in Figure 1 contain a constant term that depends on the true sample values; this constant does not affect the minimization and is only included to ease comparison with MSE. For the second experiment, we choose the SURE-optimal $\lambda$’s via two-level coarse-to-fine parameter sweeps. The SNR of the real data is 16.7 dB; we add complex Gaussian noise to reduce the SNR by factors of 2, 4, 8, and 16. The MSE and signal-to-error ratio (SER) values are computed with respect to the fully-sampled acquired data.

Results

Figure 1 compares the SURE estimate and MSE (using the acquired full k-space as ground-truth) for L1-SPIRiT reconstructions from data with 16x amplified noise for $\lambda$’s from $10^{-4}$ to 0.1. These error criteria closely match in shape, with a maximum deviation of 0.2 dB. The second experiment uses SURE-optimal $\lambda$ selection for a range of noise levels. Figure 2 shows the optimal $\lambda$’s and reconstructed image SERs for both SPIRiT and L1-SPIRiT (with the SURE-optimal $\lambda$) at these noise levels; as SNR decreases, increased regularization leads to greater improvement over un-regularized SPIRiT. Figure 3 depicts sum-of-squares reconstructed images and difference images for the 4x amplified noise case.

Discussion

These results using real data suggest that this Monte-Carlo method for automatic regularization parameter selection is effective for L1-SPIRiT reconstruction. We expect this method can be used to tune L1-SPIRiT without user intervention, which is useful in practice. Because Monte-Carlo SURE is a black-box approach that uses only the output of the reconstruction and not the structure of $f_\epsilon$, this implementation can be generalized to other parallel imaging reconstruction and denoising methods without substantial adjustment.

References


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Fig. 1: SURE nearly matches the MSE.

Fig. 2: L1-SPIRiT (green) outperforms SPIRiT (blue), while the SURE-optimal parameter $\lambda$ (red) is higher, at low SNR.

Fig. 3: Reconstructed and difference images for SPIRiT (middle) portray significant noise amplification, while the noise level in the L1-SPIRiT (right) output with SURE-optimal $\lambda$ is significantly reduced, almost matching the original image (left). The difference images are windowed up by a factor of 4.