On SNR performance of sequence designs for dynamic imaging of hyperpolarized $^{13}$C compounds

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Introduction: Hyperpolarized metabolically active substances provide a promising approach to investigating in vivo metabolism in real time. However, due to the transient lifetime of hyperpolarization and fast metabolic conversion of injected substances such as $^{13}$C labeled pyruvate, suitable fast dynamic spectroscopic imaging sequences are required. Several methods have been proposed for efficient spatiotemporal encoding for dynamic imaging of hyperpolarized compounds. The current work presents a theoretical framework to compare three frequently used pulse sequence designs for dynamic imaging of metabolic active hyperpolarized compounds. If flip angles are adapted to optimal values the SNR stays the same if the maximum BW is limited due to technical limits such as $\Delta T_E$.

Methods: Considering a given voxel size, concentration and nucleus, the SNR dependency of a single frame $n_d$ of a dynamic scan can be written as:

$$SNR(n_d) \propto \sqrt{N_{enc}} \sqrt{BW} N_p \cdot M_z(n_d)$$

where $T_1$, $T_2$ and $T_2^*$ are the relaxation time constants, $\alpha$ flip angle, $N_{enc}$ number of signal encoding steps per time frame, $N_p$ number of points acquired during readout, TR repetition time, $\Delta T_E$ time shift (multi-echo), $T_E$ echo time (gradient echo), $T_E$ echo time (spin echo) and BW bandwidth. A two compartment model is assumed for the magnetization $M_t(n_d)$. Restricting to the forward reaction from pyruvate $P(t)$ to bicarbonate $B(t)$ two differential equations result:

$$\frac{dP}{dt} = -k_{P+B} + \frac{1}{T_1} + \frac{1}{T_2} \cos(\alpha P)$$

$$\frac{dB}{dt} = k_{P+B} P(t) - \frac{1}{T_1} + \frac{1}{T_2} \cos(\alpha P)$$

SNR in every time frame and mean SNR of the first 20 dynamics were compared.

Results: Figure 2a shows the maximum SNR of pyruvate and bicarbonate dependent on the flip angle. In case of the SPSP selective excitation SNR optimal flip angles for bicarbonate were considered. The EPSI sequence provides the highest SNR for pyruvates, based on short echo times $T_E^{\text{ep}}$. For SPSP selective excitation provides highest SNR for all flip angles. With the condition that the mean SNR is the same for pyruvate and bicarbonate, an optimal flip angle on pyruvate of about $5^\circ$ was found for the SPSP selective excitation, indicated in Figure 2a, providing a 22% increase in mean SNR on bicarbonate compared to EPSI with a 40% increase at the beginning to 5% increase at time point $t = 60s$ as shown in Figure 2b. However, optimal flip angles depend on the dynamic repetition time $\Delta t$, the relaxation time $T_1$ and the number of encoding steps per dynamic $N_{enc}$. Undersampling techniques reducing $N_{enc}$ do not automatically lead to a decrease in SNR in contrast to conventional imaging. If flip angles are adapted to optimal values the SNR will stay the same. For single shot readouts the situation is similar. SNR stays the same if the BW is adapted to the reduced number of points $N_p$, so that the acquisition time is kept constant. However, if the maximum BW is limited due to technical limits such as maximum slew rate or gradient strength, acceleration techniques allow enhanced image resolution with optimal SNR as shown in Figure 2c.

Discussion: This work presents a theoretical framework for SNR comparison of sequences for dynamic imaging of metabolic active hyperpolarized substances. For metabolic products SPSP and multiband excitation can be considered most optimal in terms of SNR. Single shot methods provide very short acquisition times, which are beneficial specially in cardiac experiments and should be preferred over phase encoded EPSI sequences. However, high demands on the gradient system may limit the use of SPSP and multiband excitation. In contrast to conventional imaging, undersampling techniques do not automatically decrease SNR compared to the fully sampled case. Undersampling may be beneficial compared to the full sampling, if BW is limited for technical reasons.