**4 Dimensional, Single Step Laplacian Algorithm for Phase Unwrapping in 4D MR Flow**

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**INTRODUCTION** Robust, automated methods are needed to remove phase aliasing from velocity encoded MRI data, especially for 4D flow MRI, where manual correction is impractical. Phase unwrapping is a well studied problem in 2D and 4D PC MRI, yet a notoriously difficult task in-vivo due to low SNR regions and complex flow patterns. However, with high spatial and temporal resolution 4D-flow, phase continuity occurs both in 3D space and time, providing significantly more information for phase unwrapping [1]. This study introduces a Laplacian based single-step phase unwrapping algorithm for volumetric phase maps [2-3], to flow imaging for the first time, and investigates the added benefit of increasing the algorithm’s domain to include temporal continuity.

**ALGORITHM** Since MR encodes velocity information into phase, there is an intrinsic dynamic range of $-\pi$ to $\pi$. The true phase of the data $\phi(r)$ can be represented as: $\phi(r) = \phi_w(r) + 2\pi n(r)$, where $\phi_w(r)$ is the acquired wrapped data, and $n(r)$ is an integer. We can calculate an estimate of $n(r)$ with the Laplacian $(\nabla^2)$ of the true and wrapped phases: $n(r) = \frac{1}{2\pi} \nabla^2 (\nabla^2 \phi(r) - \nabla^2 \phi_w(r))$. The Laplacian of the true phase is estimated based on an assumption of continuity, so that all gradient terms are minimized by rounding by $2\pi$. This is calculated from the wrapped phase by: $\nabla^2 \phi = \cos \phi_w \nabla^2 (\sin \phi_w) - \sin \phi_w \nabla^2 (\cos \phi_w)$. Both the Laplacian and inverse Laplacian $(\nabla^{-2})$ can be calculated quickly using Fast Fourier Transform (FFT) based solvers, essentially reducing the complexity of the algorithm to 4 FFTs over the entire dataset. Two algorithms were implemented: a 3D Laplacian enforcing phase continuity across the 3 spatial domains and a 4D Laplacian that also enforces consistency in the temporal phase evolution.

**METHODS** The algorithms were tested on time-resolved, volumetric PC MRI from numerical phantoms and in vivo datasets. The phantom consisted of three different sized vessels with realistic spatial and temporal flow profiles. The unwrapping algorithms were tested under various VENC and SNR conditions and the number of voxels that had phase errors after the reconstruction was counted. The unwrapping schemes were subsequently applied to in vivo whole chest datasets acquired in volunteers with a radially undersampled 4D flow acquisition, PC VIPR [4]. Parameters include: isotropic spatial resolution of 1.25 mm, imaging volume: 32 x 32 x 16 cm, 27 cardiac phases with a temporal resolution of 29 ms, 10 min total scan time, VENC = 80 cm/s.

**RESULTS** Figure 1 shows unwrapping outcomes as a function of VENC and SNR. For both algorithms, very low VENC values led to unrecoverable aliasing. The 4D Laplacian successfully unwrapped aliased voxels for lower VENC data than the 3D version of the algorithm. Figure 2 shows an example of such a case where only the 4D algorithm was successful. Representative in vivo results in Figure 3 show unwrapping in cases of moderate wrapping in the aorta. Unwrapping was successful for both 4D and 3D algorithms, results are shown for the 4D version. The computational time (size = 256x256x256x27-34) with 24 threads@2.3GHz was 6-8 minutes per flow encoding direction.

**DISCUSSION** The method presented is a simple and reliable single step method for unwrapping 4D-flow data. The method improves upon previously published versions of the algorithm by utilizing all 4 dimensions of the phase field. Computation is quick and requires no outside parameters or masks, making it easy to integrate into a reconstruction without the need for any human guidance. While adding boundary conditions or other region based approaches might improve the results, the ease of use and reliability would be compromised.

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Fig 1: Charts showing unwrapping in the phantom for different VENC and SNR values. Cases that were completely unwrapped are set to black. Max velocity = 100 cm/s

Fig 2: Phase images from the phantom around peak systole (SNR = 10, VENC = 40 cm/s, max velocity = 100 cm/s). All wraps are corrected in the large vessels, but only the 4D corrects the small vessel (red arrow).

Fig 3: Images from before and after unwrapping in the aorta of two volunteers with low SNR images. Almost all wrapped voxels are seen to be corrected after applying the 4D Laplacian operator.