GENERAL CLOSED-FORM EXPRESSIONS FOR DKI PARAMETERS AND THEIR APPLICATION TO FAST AND ROBUST DKI COMPUTATION BASED ON OUTLIER REMOVAL

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INTRODUCTION AND PURPOSE
Non-Gaussianity quantification of water diffusion through diffusional kurtosis imaging (DKI) [1] attracts clinical researches of its application to short paper, we first show that stationary points can be used, that is, computation error often and abnormal tissues [2]. One of the three b-values [6], DKI computation can lead us to a practical range; 0<. Validationerror voxels with values out of the range of (D) measured in diffusion weighted imaging. For minimizing J with regard to KD, and S

THEORIES AND METHODS
General closed-form expressions: The least square fitting in DKI can be rewritten as a minimization problem of an objective function J below.

\[ J = \sum_{\alpha=1}^{N} \left( \frac{1}{6} b_\alpha^2 D^2 K - b_\alpha D - \ln \left( \frac{S_{\alpha}}{S_0} \right) \right)^2 \]

Note that we have N kinds of b-values (\(a=1\ldots N\)) and corresponding signal values \(S_{\alpha}=S(b_\alpha)\) measured in diffusion weighted imaging. For minimizing J with regard to KD, and S

\[ D = \frac{(A_{10} A_{40} - A_{42} A_{30}) A_{31} - (A_{20} A_{42} - A_{30}) A_{32} - (A_{10} A_{20} - A_{30}) A_{21}}{2 A_{10} A_{20} A_{30} + A_{20} A_{40} - A_{42} A_{30} - A_{30}^2}, S_0 = \exp \left( \frac{A_{10} A_{40} - A_{42} A_{30} + A_{20} A_{40} - A_{42} A_{30}}{A_{40} - A_{20}^2} \right), \text{and} K = 6 \frac{A_{30} D^{-1} + (A_{10} A_{20} - A_{30}^2) D^{-2}}{A_40} \]

where \(A_{ij} = \left( \sum_{\alpha=1}^{N} b_\alpha^{i+J} \cdot (\log S_{\alpha})^j \right)/N\), such as \(A_{40} = \left( \sum_{\alpha=1}^{N} b_\alpha^4 \right)/N\) and \(A_{21} = \left( \sum_{\alpha=1}^{N} b_\alpha^2 \cdot \log S_{\alpha} \right)/N\), for the instances.

Simultaneously, we can obtain the minimum value of the objective function; \(J_{\text{min}}\) as a closed-form expression consisting of \(A_\beta\) and \(N\) as following.

\[ J_{\text{min}} = \text{N} - 2 A_{20} A_{40} A_{21} - A_{40} A_{41}^2 - A_{21}^2 \frac{\left( A_{10} A_{40} - A_{42} A_{30} - A_{20} A_{40} + A_{42} A_{30} \right)^2}{\left( A_{40} - A_{20}^2 \right)^2} \left( A_{10} A_{20} - A_{30}^2 \right) \left( A_{30} A_{30} - A_{20}^2 \right) \]

Because J is a summation of square errors at sample pairs of b-value and signal (\(b_\alpha S_{\alpha}\)), \(J_{\text{min}}\) expresses the mean square error (MSE) per sample pair. In addition to the expressions including \(S_0\) above, we can obtain the expressions of \(D\) and \(K\) in a similar way for the case we trust measured \(S_0\) values.

DKI computation with outlier removal: In DKI images obtained from clinical data, voxels of extraordinary values due to computation error often appear as salt-and-pepper noise. Those values of \(D\) and \(K\) are often lower than the general lower bounds such as \(D>0.0\) and \(K>2.0\) on physical bases. In addition, it was reported that \(K\) values in brain are around 1.0 [1], which can lead us to a practical range; 0<K<2.0. When we have certain amount of sample pairs (\(b_\alpha S_{\alpha}\)), it is effective to remove outliers to reduce fitting error \(J_{\text{min}}\). The RANSAC algorithm [5] is based on an idea of random sample removal to search better fitting. Because we have very limited number of samples, we can examine all combinations to find the least MSE instead of random combination. For instance, we only have to try \(\text{N}^C \text{N}\) times when single sample pair is removed. Because of the closed-form expressions, this algorithm is feasible enough for clinical data and standard PCs.

RESULTS AND DISCUSSIONS

Materials: Six brain DWI datasets of healthy volunteers were obtained by a clinical 3T MR scanner (Philips Achiva) with the acquisition parameters; TR/TE=3000/80 msec., 128x128 matrix, 6~32 MPG directions, seven b-values of 0, 500, 1000, 1500, 1500, 2000, and 2500 sec/mm². After removal of the bias due to Rician noise [6], DKI computation was performed with 4 methods from combinations of with/without \(S_0\) estimation (wS0 or woS0) and with/without single outlier removal (wO or woO).

Experiments 1 (computation time): Based on the closed-form expressions, we achieved over 30 times faster computation speed than the LM-based method. Because values of \(A_\beta\) are independent of DWI signals when \(j=0\), most part of the closed-form expressions can be commonly precomputed, which bring us further efficient DKI computation.

Experiments 2 (computation error ratio): For the DWI sets, we counted computation error voxels with values out of the range of (D>0, 0<K<2) in the presegmented brain mask. As shown in Fig.1, our algorithm with \(S_0\) estimation and outlier removal achieved the lowest error ratio, especially in the K value computation. We observed that most of the removed samples were mainly from higher b-values. This is natural because higher b-values yield signals with lower signal-to-noise ratio in DWI. These results clearly show the feasibility and the effectiveness of our method applied in clinical DKI.

REFERENCES