INTRODUCTION: Quantitative Susceptibility Mapping (QSM) is used to quantify tissue magnetic susceptibility, leading to applications such as tissue contrast enhancement [1], venous blood oxygenation [2], and iron quantification [3]. Quantification of the susceptibility distribution \( \chi \) involves removal of background effects on the MRI signal phase and the solution of an ill-posed inverse problem describing the mapping from the phase to the tissue susceptibility. In this work, background removal is achieved by using the effective dipole fitting algorithm [4] and susceptibility inversion is performed via imposing \( \ell_1 \) norm regularization on the spatial gradients of \( \chi \) [5]. As both algorithms are computationally demanding, it is crucial to increase the computational throughput and make regularized QSM a feasible and real-time methodology. Herein, the computational power Graphics Processing Cards (GPUs) is utilized to greatly accelerate the processing times, and both MATLAB and GPU libraries of the regularized QSM method are made available online for reproducibility (web.mit.edu/berkin/www/software.html).

THEORY: The effective dipole fitting algorithm [4] aims to estimate the background susceptibility distribution that optimally matches the field inside the region of interest (ROI), and removes this contribution to recover the foreground field map. This is achieved by solving a least-squares problem \( \chi_{\text{out}} = \arg \min_{\chi} \| M(\delta - F^c \chi) \|_2^2 \) where \( M \) is the mask that marks the ROI, \( M_i \) is the complement of \( M_i \), \( \delta \) is the field map, \( F \) is the Fourier transform matrix, \( D \) is the susceptibility kernel in \( k \)-space, and \( \chi \) is the magnetic susceptibility. After solving this least-squares problem, the field map generated only due to the internal local effects is found by \( \delta_{\text{in}} = \delta - F^c \chi_{\text{out}} \). The second part of the problem is solving for the tissue susceptibility distribution \( \chi_{\text{m}} \) that satisfies \( \delta_{\text{in}} = F^{-1} D \chi_{\text{m}} \). As the susceptibility kernel \( D \) undersamples frequency components of \( \chi_{\text{in}} \) on a conical surface in \( k \)-space, the inversion benefits from regularization. Here, \( \ell_1 \) norm regularization on the spatial gradients of the susceptibility is applied, which requires the solution of the convex program \( \chi_{\text{m}} = \arg \min_{\chi} \| M(\delta_{\text{in}} - F^{-1} D \chi) \|_2^2 + \lambda \| G \chi \|_1 \). Here, \( \| G \chi \|_1 \) is the three dimensiona gradient operator and \( \lambda \) is a regularization parameter that trades off data consistency and spatial smoothness. Both systems are solved iteratively with the conjugate gradient algorithm.

MATERIALS AND METHODS: CUDA Toolkit 4.0 and CUDA developper driver 270.81 were used to enable interfacing with the GPU card Tesla C1060. Two GPU libraries to execute Fast Fourier Transform (FFT) were tested and compared: i) CUDA FFT, which is part of CUDA Toolkit 4.0; and ii) the Nakada FFT library which is an Auto-Tuning 3-D FFT Library for CUDA GPUs [6]. The GPU implementation was tested in vivo data acquired with susceptibility-weighted 3D SPGR at 1.5T (TR/TE=58 ms/40 ms, 256x256x62 matrix, 24x24x15.5 cm<sup>3</sup> FOV). The data were zero padded in all three dimensions in order to obtain linear convolution with \( D \) in \( k \)-space. The optimal regularization parameter for \( \ell_1 \), regularized QSM was found to be \( \lambda = 2 \times 10^{-4} \) using L-curve method [7].

RESULTS: For both parts of the algorithm, we were able to get an output similar to one when the code is executed on MATLAB. The RMSE (root mean square error) is 5% between MATLAB and the GPU result for the background removal algorithm. The QSM algorithm on GPU has a RMSE less than 1% when compared to MATLAB result. However, when the GPU background removal output is used as an input for the GPU QSM algorithm, the background removal algorithm error propagates into the QSM algorithm causing a RMSE of 7% in \( \chi \). Double precision was used in MATLAB computations and single precision for GPU. As FFTs constitute the main computational burden, the speed up obtained relative to the MATLAB implementation was seen to depend on the FFT library of choice. While the employing CUDA FFT library yielded a 2-fold acceleration, using the optimized Nakada FFT library resulted with a speed up by a factor of 20. The total reconstruction time was 160 seconds on the GPU, compared to the 60 minutes processing time with MATLAB running on a workstation with 8 processors and 24 GB of memory. GPU reconstructed susceptibility map for the in-vivo study is presented in Fig. 1. The difference of the average \( \chi \) values for the 6 regions of interest (ROIs) between the GPU reconstruction and MATLAB reconstruction is shown in Fig. 2.

CONCLUSION: The significant acceleration by factor of 20 in background field removal and regularized QSM obtained with the GPU implementation may play an important role towards real-time, online reconstruction of high-fidelity susceptibility maps.