Optimality of Equally-Spaced Phase Increments for Banding Removal in bSSFP
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Introduction

The balanced steady-state free precession (bSSFP) pulse sequence is capable of high-SNR imaging in relatively short scan times. However, the off-resonance sensitivity can lead to signal nulls and undesirable banding artifacts in the images. Common methods for eliminating or reducing these banding artifacts involve the acquisition of multiple images with different RF phase increments to shift the bands, and subsequent image combination [1-4] or parameter estimation [5-6] techniques. Virtually all of these techniques utilize phase increments (Δθ) that are distributed evenly between 0 and 2π and here we restrict the investigation to four phases, as commonly used. Figure 1 shows the resulting spectral profiles in the case of four equally spaced phase increments. While equally spaced phase increments seem intuitive because this maximizes the distance between signal nulls, to our knowledge, there has been no previous work to show that this choice is optimal for minimizing signal variations due to off-resonance effects. In this work, we formulate bSSFP banding removal as a parameter estimation problem, and we use the Cramér-Rao bound of the variance of the parameter estimates for any unbiased estimator. It is therefore of interest to minimize, in some sense, the CRB with respect to the experimental setup. Here, we consider finding the optimal vector of phase increments (Δθ) for the acquisition. We resort to a numerical evaluation of the CRB due to the high complexity of the analytical expression. Because the CRB is a function of T1, T2, off-resonance (θ), flip angle, and KMo which are specific to each scan, we suggest a worst-case scenario optimization. The approach is to minimize the maximum CRB of KMo over all θ, T1, T2, α, and KMo. However, empirically it seems that the optimal Δθ only depend on θ, and thus, assuming that θ is unknown, it is sufficient to minimize the CRB over all θ ∈ [0, 2π],

\[
\Delta \theta_{opt} = \arg \min_{\Delta \theta} \max_{\theta \in [0, 2\pi]} CRB_{KMo}(\Delta \theta, \theta).
\]  

(Eq. 2)

Methods

Given that the optimal phase increments only depend on θ, we can keep the remaining parameters fixed. Due to the many local minima of the problem in Eq. 2, the CRB function is gridded over all phase increments Δθ and θ, and a 5D exhaustive search is conducted.

Results

We show that when using four phase increments, the optimal values from a minimax point of view are uniformly spaced: Δθ_{opt} = [0, π/2, π, 3π/2] + C, where C is any constant. Figure 2 shows that by restricting the range of Δθ-values to less than 3π/2, the CRB increases. For a range larger than 3π/2 the CRB is constant and the optimum point is Δθ_{opt} = [0, π/2, π, 3π/2] + C, where C is any constant.

Discussion

The result might be intuitive, since a periodic signal with an unknown phase shift is measured: it gives a uniformly distributed signal power in terms of probability, and maximizing the distance between the phases minimizes their correlation. However, in general, uniform sampling does not need to be worst-case optimal, and it is therefore important to show that these acquisition phases have theoretical support. It is interesting to note that if prior information regarding θ is available, finding a non-uniform spacing of the phases that gives a lower CRB is possible, but the performance gain is likely to be small. Although the optimization goal was to minimize the maximum CRB of KMo, the equally spaced phases also minimize the variance of the other model parameters, meaning that the choice is not application-specific.

Conclusion

We showed that the (1) worst-case performance in terms of the CRB is minimized when using equally spaced phases, and (2) the resulting performance is relatively close to the optimal performance, had the true value of θ been known a priori.

References


Figure 1. Spectral profiles of four bSSFP sequences with phase cycling Δθ = 0 (dashed red), Δθ = π/2 (solid yellow), Δθ = π (dotted green), and Δθ = 3π/2 (dotted blue). Phase cycling yields translation of the magnitude profiles (top) and phase profiles (bottom), with additional phase offsets in each of the phase profiles.

Figure 2. Minimax objective value (Eq. 1) as the allowable range of Δθ (i.e., Δθ_{max} - Δθ_{min}) is increased from 0 to 2π. The minimum is attained when the allowable Δθ range is 3π/2, and the optimal point is Δθ_{opt} = [0, π/2, π, 3π/2] + C, where C is any constant.

Figure 3. Estimated distribution of CRB values, normalized with min(CRB), for all possible phases Δθ and off-resonances θ, together with the worst-case CRB over all θ for equally spaced phases.