Greater Acceleration through Sparsity-Promoting GRAPPA Kernel Calibration

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Introduction
Auto-calibrating accelerated parallel imaging methods like GRAPPA [1] or SPIRiT [2] rely on acquiring a full field of view (FOV) block of k-space called the ACS lines to train the interpolation or consistency kernels. When an insufficient number of ACS lines are acquired, or those samples have low signal-to-noise ratio, the reconstruction quality suffers from residual aliasing and/or greater noise amplification. Because GRAPPA requires a reasonably large kernel to reconstruct images from highly undersampled data, acquiring sufficiently many ACS lines to yield a high quality image may limit the total effective acceleration. We are able to achieve. Regularization techniques like Tikhonov regularization or constraining the GRAPPA kernel to act like a frequency shift operator [3] may mitigate the tradeoff between reconstruction quality and effective acceleration. Instead, we develop a sparsity-promoting calibration method for GRAPPA initially proposed in [4] and explore the effects of Tikhonov regularization and sparsity-promoting calibration on the tradeoff between image quality and total acceleration.

Theory
Given ACS lines \( d_{\text{ACS}} \), the GRAPPA kernel fit equations \( y_{\text{src}} = Y_{\text{acs}} g \) for least-squares kernel calibration are formed as is usual, and the kernel \( g \) is calibrated using both the fit equations and an \( \ell_2, \ell_1 \) norm-based joint sparsity regularizer [4]:

\[
\minimize \| y_{\text{src}} - Y_{\text{acs}} g \|^2 + \lambda \| \Psi Y g - d \|_2
\]

where \( N \) is the total number of fits, \( \lambda \) is a tuning parameter, \( \Psi \) is the DWT sparsifying transform, \( F^{-1} \) is the inverse DFT, and \( \text{GRAPPA}(g,d) \) is the GRAPPA reconstruction operation using kernel \( g \) and acquired data \( d \). For uniformly spaced Cartesian subsampled k-space, the GRAPPA reconstruction can be expressed in terms of convolution operations, yielding an affine function of \( g \) and a linear adjoint operator that are straightforward to evaluate. As with conventional GRAPPA, the acquired data \( d \) is unchanged in the result; only the kernel and the interpolated k-space are affected by the joint sparsity prior we impose.

Methods
The coil array noise covariance is measured with a fast noise-only acquisition (no RF excitation), and the reference image is acquired using a T1-weighted MPRAGE sequence (256×256×176 voxels at 1.0 mm isotropic resolution), requiring 8 minutes in a Siemens Trio 3 T scanner with a vendor-supplied 32-channel head array receive coil. The slice shown in Figure 1(a) is transverse to the frequency-encoded direction. It is cropped and uniformly undersampled by a factor of 4 in both phase-encoded directions to emulate 2-D acceleration. Full-FOV ACS data is retained, too, yielding total acceleration factors of 10 and 13 for 36×36 and 24×24 ACS blocks, respectively. The tuning parameter \( \lambda \) is selected manually via a parameter sweep. Image quality is evaluated using difference images between magnitude images. Magnitude images are generated from the coil images using un-accelerated SENSE [5], which requires the coil array noise covariance matrix and low-resolution estimates of the receive coil sensitivities from ACS lines, apodized to reduce Gibbs ringing. Peak signal-to-noise ratio (PSNR) is included as a quantitative metric (although PSNR is not always representative of perceived image quality). In Figure 2, the PSNR trend is plotted versus total acceleration with un-regularized, Tikhonov-regularized, and sparsity-promoting kernel calibration.

Results
Two cases are investigated: (i) the kernel source points (512) outnumber the ACS fit equations, and (ii) there are enough ACS lines to perform a fit without regularization. For the first, the images in Figure 1(b-c) depict differences between Tikhonov and sparsity-promoting regularization with 24×24 ACS lines. In this range, sparsity promoting calibration more effectively mitigates aliasing. The second case is evident in Figure 1(d-f) with 36×36 ACS lines: either regularization reduces noise amplification, but residual aliasing remains in the GRAPPA result with Tikhonov-regularized calibration. The trends in Figure 2 suggest sparsity-promoting calibration maintains consistently higher PSNR than un-regularized and Tikhonov-regularized GRAPPA over a broad range of accelerations.

Discussion
The sparsity-promoting GRAPPA kernel calibration technique yields un-aliased images, even when few ACS lines are available. The proposed technique effectively shifts the image quality vs. acceleration tradeoff, enabling high quality reconstructions using fewer ACS lines for calibration.

References