On the Optimum Source Impedance for MRI Phased Array Coils
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Introduction
Typically phased array coil elements are impedance matched to a source impedance of 50 Ω using a lumped-element quarter wave transformer circuit comprising of a matching capacitor $C_m$ and matching inductor $L_m$. Each element is subsequently mismatched to the low input impedance of a preamplifier, typically < 5 Ω, for preamplifier decoupling (1). The choice of 50 Ω source impedance seems to be governed by optimisation of the preamplifier noise figure, preamplifier gain and the availability of 50 Ω RF components and measurement equipment. Recent advances in CMOS low noise amplifiers suggest that lower noise figures can be obtained at source impedances greater than 50 Ω (2). The aim of the present work is to explore the issue of source impedance for optimum phased array performance based on preamplifier decoupling, preamplifier noise figure and preamplifier gain.

Analysis

Impedance Matching & Preamplifier Blocking Impedance: A phased array coil element may be represented by the equivalent circuit shown in Fig. 1, where $R_i$ is the input impedance of the preamplifier (Ω), $R_s$ is the source impedance of the RF system (Ω) and $\omega$ is the Larmor frequency (rad/s). The match inductance $L_m$ (H), match capacitance $C_m$ (F) and preamplifier blocking impedance $Z_{block}$ (Ω) can be calculated using Eqs.[1-3] (1).

![Figure 1. Circuit model of a phased array coil element.](image1)

Preamplifier Decoupling: A pair of coupled identical phased array coils may be represented by their equivalent circuit models shown in Fig. 2, where $k$ is the coupling coefficient. The degree of decoupling can be determined by Eq.[4] (3,4).

Preamplifier Noise Figure & Gain:
The minimum noise factor for a CMOS common-source low noise amplifier (LNA) is given by Eq.[5]. $Q_{opt}$ is given by Eq.[6], where $\omega_0$ is the operating frequency, $\alpha_T$ is the unity current gain (cut-off) frequency, $c = 0.395$ is the gate and drain noise current correlation coefficient, $\alpha = g_{dr}/g_{ds}$, $g_m$ is the device transconductance, $g_{ds}$ is the zero-bias drain conductance and $\gamma$ and $\delta$ are coefficients of channel and gate induced noise respectively (2.5). $Q$ is also given by Eq.[7], where $C_g$ is the gate-source capacitance and $R_i$ is the source impedance (5). The effective transconductance of the common-source LNA under perfect matching conditions is given by Eq.[8] (5).

$$L_m = \frac{1}{\omega \sqrt{R_s R_i}} \quad [\text{Eq.1}]$$

$$C_m = \frac{1}{\omega \sqrt{R_s R_i}} \quad [\text{Eq.2}]$$

$$Z_{block} = Q^2 R_p = \frac{\omega_0^2}{R_i} \quad [\text{Eq.3}]$$

$$F_{min} = 1 + \frac{\gamma}{\alpha} \frac{2\delta \alpha^2}{5\gamma} Q_{opt} \quad [\text{Eq.5}]$$

$$Q_{opt} = \sqrt{1 + 2\frac{\gamma}{\alpha} \left(\frac{5\delta}{\alpha^2} + \frac{5\gamma}{\alpha}\right)} \quad [\text{Eq.6}]$$

$$Q = \frac{1}{\alpha C_g R_i} \quad [\text{Eq.7}]$$

$$G_{n,eff} = \frac{1}{2R_i} \left(\frac{\omega_0}{\alpha_T}\right) \quad [\text{Eq.8}]$$

Discussion
Preamplifier blocking impedance and hence preamplifier decoupling improve with increasing $R_i$, as shown by Eqs.[3,4]. For CMOS LNA’s Eqs.[5-7] demonstrate that the minimum noise factor $F_{min}$ improves with increasing $R_s$. However, the price paid for achieving a minimum noise factor by increasing $R_s$ is a reduction in the effective transconductance and hence gain of the preamplifier, as described by Eq.[8]. This may appear at first inspection to be a high price to pay, but adequate gain can be achieved by cascading gain stages, and it is well known that the noise factor of the first gain stage dominates the overall noise factor of a cascaded amplifier (6). Clearly experimental validation is required to support the case for moving to a higher source impedance, but it is hoped that the arguments presented here will lead researchers to explore the optimum source impedance and semiconductor technology for MRI phased array LNA’s. Finally, the authors note that the noise factor improvements afforded by increasing source impedance for CMOS LNA’s may also be afforded by GaAs FET LNA’s as suggested in Fig. 1a of ref (7).

References
1. Roemer et al. The NMR phased array. MRM 16,192-225 (1990)