INTRODUCTION: MRI reconstruction from undersampled k-space data requires regularization to reduce artifacts and improve image quality. Nonquadratic regularizers, e.g., edge-preserving ones or those based on the l-norm, have proven to be useful in MRI [1], but successful application of such criteria depends on proper selection of the regularization parameter (\( \lambda \)) that controls the degree of smoothing imposed on the reconstruction. Several quantitative methods are available for automatic selection of \( \lambda \) such as the discrepancy principle (DP) [2], the L-curve method (LCM) [3], generalized cross-validation (GCV) [4] and the estimation of mean-squared error (MSE) type measures [5]. DP and LCM are able to handle a variety of nonlinear reconstruction algorithms, but can lead to over-smoothing [2] or become sensitive to changes in \( \lambda \) [3], respectively. GCV is a popular choice in reconstruction problems especially involving linear algorithms [4]. In the linear case, GCV is simple to implement and can provide asymptotically optimal selection of \( \lambda \) [4]. MSE-type estimates are attractive alternatives to GCV as such estimates can provide (near) optimal results even in the nonasymptotic regime [5]. However, both GCV [6] and estimation of MSE-type measures become nontrivial and computationally involved for nonlinear algorithms. In this work, we propose a practical means of computing GCV and an MSE-type estimate for nonlinear MRI reconstruction using the split-Bregman algorithm [7]. We illustrate with experiments on real MR data that they can be employed for near-optimal adjustment of \( \lambda \) for reconstruction from undersampled Cartesian k-space data using nonquadratic regularization.

METHODS

MRI Reconstruction: We perform MRI reconstruction by minimizing a cost function:
\[
\min_{u} \frac{1}{2}||y - Au||^2 + \lambda \kappa(\Psi u),
\]
where \( y \) is the measured data, \( A \) denotes the linear operator, \( \kappa(\Psi u) \) represents the L1 regularization penalty, \( \lambda \) is the regularization parameter, and \( \Psi \) represents the nonlinear operator used for reconstruction. The specific form of \( \Psi \) depends on the algorithm used for reconstruction. In the sequel, we show how to evaluate \( \lambda_{opt} \) analytically for the SB algorithm.

Experimental Setup: We perform MRI reconstruction by minimizing a cost function:
\[
\min_{u} \frac{1}{2}||y - Au||^2 + \lambda \kappa(\Psi u),
\]
where \( y \) is the measured data, \( A \) denotes the linear operator, \( \kappa(\Psi u) \) represents the L1 regularization penalty, \( \lambda \) is the regularization parameter, and \( \Psi \) represents the nonlinear operator used for reconstruction. The specific form of \( \Psi \) depends on the algorithm used for reconstruction. In the sequel, we show how to evaluate \( \lambda_{opt} \) analytically for the SB algorithm.

Evaluating \( \lambda_{opt} \): We recursively evaluate \( \lambda_{opt} \) using linearity-, product- and chain-rule for Jacobian matrices [9]: As \( v \) and \( w \) are functions of \( y \) (via \( u \) in the SB algorithm, we get
\[
\lambda_{opt} = \min_{\lambda} \frac{1}{2}||y - Au||^2 + \lambda \kappa(\Psi u).
\]
Applying chain rule [9], \( \lambda_{opt} = \min_{\lambda} \frac{1}{2}||y - Au||^2 + \lambda \kappa(\Psi u) \).

CONCLUSIONS: We demonstrated the feasibility of using GCV and MSE-estimation for automated adjustment of the regularization parameter \( \lambda \) for nonlinear MRI reconstruction using the split-Bregman (SB) algorithm. GCV and MSE require the trace of a linear transformation of the Jacobian matrix \( \kappa(\Psi u) \) that we estimated stochastically and iteratively for the SB algorithm. We illustrated with experiments on real MR (phantom) data that GCV and MSE are able to provide near-MSE-optimal selection of \( \lambda \). The techniques discussed here can also be extended, in principle, to other regularization criteria, reconstruction algorithms, and non-Cartesian and parallel MRI.

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