Joint Inference of Field Inhomogeneities with Fat Likelihood Estimation from Three Echoes

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Introduction: Robust multi-echo water-fat separation at high field remains challenging in the presence of long echo-spacing and large field inhomogeneities. Recent work by Yu et al. [1] greatly improves the robustness of field map estimation by computing a priori fat likelihood. However, obtaining reliable fat likelihood requires rather long echo train (6 echoes) to observe the signal difference arisen from single-peak water model and multiple-peak fat model. In this work, we demonstrate that inconclusive fat likelihood estimation from three echoes can be effectively combined with field map smoothness via a joint inference algorithm, which enables robust water-fat separation in challenging imaging scenarios at 3T.

Theory: The signal $S_i$ received at echo time $TE_i$ is

$$S_i = (W + F)\sum a_i e^{i2\pi f_i(TE_i)}e^{i2\pi f_w i},$$

where $\Delta f_i$ are the chemical shift between water $W$ and the $i^{th}$ spectral component of fat $F$. Given the multi-peak fat parameters $a_i$ and $\Delta f_i$, we can compute the least-squares fitting error $J_W$ as the function of the unknown field inhomogeneity $\psi$ [2]. Two smallest local minima of $J_W$ at $\psi_W$ and $\psi_F$ are located within one spectral FOV [0, 1/ΔTE], where $\psi_W$ and $\psi_F$ result in water-dominant and fat-dominant separation results, respectively. Denote their corresponding fitting errors as $J_W$ and $J_F$; a priori fat likelihood $P_F$ is given by $J_W/\{J_W + J_F\}$.

The larger the water-fitting error $J_W$ as compared to the fat-fitting error $J_F$, the more likely the fat is dominant (Fig. 1a). However, $J_W$ and $J_F$ can be very close when estimated from three echoes using the multi-peak fat modelling, especially for voxels containing water-fat mixture (Fig. 1b). Similar $J_W$ and $J_F$ results in inconclusive fat likelihood (i.e., $P_F \approx 0.5$), which imposes challenges on resolving the correct field inhomogeneity.

Methods and Results: For each voxel, $\psi_F$ and $\psi_W$ and their spectral replicas are considered feasible field map values, which are assigned with the corresponding a priori likelihood $P_F$ or $1 - P_F$. An interaction potential $V_{pq}$ is exchanged between two neighboring voxels $p$ and $q$ on their feasible field map values $\psi_p$ and $\psi_q$:

$$V_{pq} \propto P_{q} P_{p} \exp(-|\psi_p - \psi_q|^2),$$

which blends both the difference between two feasible values and their a priori likelihood. The interaction potential between $\psi_p$ and $\psi_q$ is large when both a priori likelihoods are close to 1 and the difference between two values is small. More importantly, the interaction potential enables a voxel with strong a priori likelihood to exert influence on its neighboring voxels with less conclusive a priori likelihood. The interaction potential is embedded in a soft-decision message from $p$ and $q$ at the $i^{th}$ iteration

$$m_{q→p}(\psi_p) = \sum_{\psi_q} (V_{pq} \prod_{x \in \text{nei}(p)} m_{x→p}(\psi_q)) \eta^{-1}_{q→p}(\psi_p),$$

where $\eta(p) \setminus q$ denotes the set of pixels neighboring to $p$ other than $q$. By exchanging the soft-decision messages between voxels, the algorithm jointly infer the most likely field values across the whole image grids [3].

A multi-echo SPGR sequence was used to perform liver studies on 11 obese cats using a body matrix coil and a spine coil on a Siemens Tim Trio 3T scanner. The images were acquired with respiratory gating and 2 ms ΔTE, 5° flip angle, 128x128 matrix size, 20x20 cm² FOV. Fig. 2a shows that subcutaneous fat has very strong fat likelihood estimate (i.e., $P_F \approx 1$), while some regions in the fatty liver have inconclusive fat likelihood estimates ($P_F$ fluctuates around 0.5 inside the black circle). In spite of inconclusive fat likelihood estimates, the proposed inference algorithm achieves uniform water-fat separation for all studies. The sample separation results are shown in Fig. 2b and c.

Conclusion: This work presents a joint inference algorithm which effectively combines the field map smoothness constraint and the fat likelihood estimated from three echoes. This technique is potentially useful for motion-sensitive applications, such as liver fat quantification.