Improving edge recovery in undersampled MRI reconstruction

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Introduction

In MRI, significant aliasing artifacts are introduced when the k-space is highly undersampled, which blur the reconstructed edges. The edges usually contain useful morphological information for the diagnosis of focal areas, such as the margin of the tumor, and the caliber of the vessel. Therefore, undersampled MRI reconstruction with good edge recovery is important for clinical diagnosis. In this work, we define a weighting matrix from an edge scoring function to obtain better edge recovery. The edge scoring function is designed by considering the strength, orientation, and spatial continuity of the edge features in local patches in à trous wavelet domain. Simulations indicate that the proposed method yields reconstructions with better edge recovery than conventional compressed sensing methods, and requires fewer k-space measurements to achieve reconstruction error of the same level.

Methods

The reconstruction formulation for the proposed method is \( \min_x \| \mathbf{y} - \mathbf{F} \Psi \mathbf{x} \|_2^2 / 2 + \lambda \| \mathbf{W} \mathbf{x} \|_1 \), where \( \Psi \) is the sparsifying transform (à trous wavelet in this work), \( \mathbf{y} \) is measured k-space data, \( \mathbf{x} \) is the sparse representation of a \( N \times 1 \) signal \( \mathbf{x} = \mathbf{\Psi} \mathbf{u} \) produced by vectorizing the magnetic resonance image. \( \mathbf{F} \) represents undersampled Fourier transform, \( \lambda \) is a regularization parameter governing the trade-off between the data consistency and sparsity. \( \mathbf{W} \) is a weighting matrix generated from the edge scoring function \( f(\cdot) \) by \( \mathbf{W} = 1 / f(\cdot) \). \( f(\cdot) \) is designed by combining two components \( f(\cdot) = f_1(\cdot) \cdot f_2(\cdot) \), where the first component \( f_1(x,y) = \sqrt{\sum \alpha_{x,y}^2} / (\cos(\Delta \theta) + 1) \) is used to quantify the contribution of wavelet coefficient \( \alpha \) to the edge features in the local patch, while the second component \( f_2(x,y) = \sqrt{(x_{x,y} - x_{x,y})^2 + (y_{x,y} - y_{x,y})^2} \) is used to measure the spatial continuity of edge features passing through \( \alpha \). \( (x_{x,y}, y_{x,y}) \) and \( (x_{x,y}, y_{x,y}) \) are two vectors defined based on the strength and orientation of the edge in a local patch, and \( \Delta \theta \) is the angle between \( (x_{x,y}, y_{x,y}) \) and \( (x_{x,y}, y_{x,y}) \). The strength and orientation of the edge is defined as follows. Suppose a square-shaped window centered at wavelet coefficient \( \alpha \), is divided into two sub-windows by a line with angle \( \theta \) (\( 0 \leq \theta \leq \pi \)), as shown in Fig. 1. Let the sum of the coefficients in the two sub-windows be \( d_1 \) and \( d_2 \). The edge strength \( d \), which reflects the intensity transition of wavelet coefficients along the edge, is defined by the maximal difference between \( d_1 \) and \( d_2 \) as \( \theta \) changes, and the angle \( \theta \) is defined as the directional orientation of the edge at \( \alpha \). Then \( (x_{x,y}, y_{x,y}) \) and \( (x_{x,y}, y_{x,y}) \) are defined by \( x_{x,y} = d \sin(\theta) \), \( y_{x,y} = \sum_{\alpha=1}^{N} \alpha_{x,y} \), \( y_{x,y} = \sum_{\alpha=1}^{N} \alpha_{x,y} \), where \( P \) denotes the local patch.

Results and discussion

The MR image used for simulations is T2-weighted and obtained from a 1.5T GE MRI scanner, as shown in Fig. 2 (a). À trous WT with spline biorthogonal filters and four decomposition levels is used as the sparsifying transform. Cartesian sampling pattern [1] with 30% measurements are acquired, as shown in Fig. 2 (b). Relative \( \epsilon_1 \) norm error (RLNE) is adopted to evaluate the normalized error presented in the reconstruction compared with the fully sampled MR image. The proposed algorithm is compared with the fast iterative soft-thresholding algorithm (FISTA) [2], reweighted \( \epsilon_1 \) norm (RWL1) minimization algorithm [3], and edge correlation incorporated algorithm (ECIA) [4]. The comparison is given in Fig. 3. We can observe that edges obtained by the proposed method are much clearer than the other three methods. The proposed method also yields lowest RLNE among the four methods. In addition, to test the performance of different methods under different sampling rates, we give the curve of RLNE versus sampling rate in Fig. 4. The image used in this trial is the same as Fig. 2 (a), with Cartesian sampling scheme employed. The curves in Fig. 4 indicates that, when the acquired k-space data grows to 25%-35%, the RLNEs for the proposed method fall between 0.04-0.07, under which we think reconstruction quality is relatively acceptable. However, RWL1 and ECIA achieve RLNEs of the same level with nearly 35%-40% k-space measurements.

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References