Rapid relative pressure map computation from velocity-encoded phase-contrast measurements

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Introduction
Spatially resolved assessment of blood pressure dynamics is desirable for the evaluation of various cardiovascular disorders. For example, the local change of pressure across vascular or valvular stenoses can be used to characterize the severity of the constriction. For the heart chambers, systolic and diastolic efficiency is directly related to the dynamics of the respective filling pressures. Time-resolved velocity data acquired by velocity encoded phase-contrast MRI can be used to derive relative pressure [1,2].

Methods
Pressure Poisson equation solver: Assuming laminar flow conditions and an incompressible Newtonian fluid with density ρ and viscosity μ, the simplified Poisson equation relates the pressure p to the velocity field V according to:

$$\nabla^2 p = \nabla \cdot (-\frac{\partial p}{\partial t} \rho \nabla V + \mu \nabla^2 V)$$  \[1\]

Equation 1 is discretized using central differences for the divergence calculations. The Laplacian is calculated using a component-wise symmetric 3-point discretization on inner voxels and a linear extrapolation on boundary voxels. The resulting system of linear equations is solved using an iterative Krylov subspace approach where in each iteration the residual norm is minimized. This so-called GMRES procedure includes an Arnoldi iteration and for each iteration step an inner least squares problem is solved. A maximum of 50 inner and 10 outer iteration steps were performed until the residual converged below a predefined tolerance.

Results
For all cardiac phases, the solver converged to a solution for all datasets. Calculation times were typically below 4s per cardiac phase on a 2.5 GHz Core Duo Intel CPU. The resulting maps showed a smooth pressure distribution across the aortic arch. In the healthy volunteers pressure distributions in the ascending aorta were larger compared to those seen in the descending aorta in early systolic phases and vice-versa in late systolic phases.

Discussion:
The proposed solver provides estimates of the relative pressure fields directly from segmented velocity field data acquired by velocity-encoded phase-contrast MRI. However, the automatic segmentation of the aorta proved difficult especially in the patient data. Segmentation errors significantly influencing the resulting relative pressure maps. Another potential problem is the excessive memory usage of the GMRES algorithm in very large computational domains. This can be addressed by an extension of the GMRES algorithm, whereby the method is restarted after a limited number of iterations. A restarted GMRES algorithm may be required for datasets with higher spatial resolution and matrix sizes. Also, appropriate preconditioning of the system may further optimize algorithm convergence and remains to be investigated.

References