Compressed Sensing in Phase-encoded Multi-dimensional Magnetic Resonance Imaging

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INTRODUCTION: Recent advances in compressed sensing (CS) [1] have accelerated MRI by exploiting signal sparsity. However, existing image recovery schemes [2] may be difficult to converge when the data matrix is large. The object of this study is to examine the applicability of CS to phase-encoded 3D MRI, which usually acquires a large data matrix with relatively long acquisition time. We propose an iterative reconstruction procedure that approximates the under-determined problem with a sequence of over-determined problems. We will demonstrate that this method is accurate, efficient and stable for recovering 3D MRI from half k-space acquisition.

THEORY: Our reconstruction scheme is based on the current CS-MRI reconstruction [2] that is solving an optimization problem (in Lagrange form) $$\min \{ |S| + \lambda \| K - \Lambda^b (\text{IFT}(S)) S^* \| \}$$ where $S$ is image, $b$ is measured phase encoding line, the IFT is 3D Fourier transform, $TV$ is total variation and $M$ is undersampling operation. However, this optimization problem is difficult to solve when image size becomes large. Hence we replace the above optimization problem by a sequence of simplified optimization problems. As shown in Fig. 1, the new reconstruction scheme relies on the optimization problem $$\min \{ |S| + \lambda \| K - \Lambda^b (\text{IFT}(S)) S^* \| \}$$ by using an iteration loop (Fig. 1a) that iteratively impose the summation of $b$ (measured phase encoding line) and $S^*$ (unmeasured phase encoding line) into $K$. The Lagrange function of inner optimization problem can be separated into two parts as $$\min \{ |S| + \lambda \| K - \Lambda^b (\text{IFT}(S)) (S^*) \| \} = \min \{ |S| + \lambda \| K - \Lambda^b (\text{IFT}(S)) \| \}$$, the first part is the Lagrange function of the original optimization problem and the second part is an additional constraint that requires the optimum to be “localized”. Hence proposed scheme generates a sequence of progressively locally optimal solutions to approximate the global optimum.

METHODS: the half k-space undersampling was achieved by randomly selecting the phase encode lines measured. The 3D acquisition order was: phase encoding in Kx-direction, phase encoding in Ky-direction and readout in Kz-direction. As shown in Fig. 3c, the sampling density function was quadratic with highest value in the center of Ky-Kz plane. The in-vivo rat brain experiment was performed in a 7T Bruker scanner. T1 contrast 3D high resolution image was acquired by a Modified Driven Equilibrium FT sequence [3] with TR/TE = 9/3 ms, Inversion Time = 1.2s, matrix size = 160×160×80, FOV = 32×32mm, slice thickness = 0.2mm, and NEX = 4. The 50% random sampling was performed retrospectively to the fully sampled dataset. The raw data was the zero filled to 256×256×80 before reconstruction.

RESULTS: In Fig.2, the value of Lagrange function of inner optimization is reduced after each inner loop and appears to approach convergence when the outer loop terminates in 20 steps. In Fig. 3b, the reconstruction results (50% k-space dataset) match up well with the original (100% k-space dataset). As shown the histograms of Fig. 3d, the error is less than 1/10 the signal of the subject. The error/image ratio is close to Gaussian with mean = -0.015 and standard deviation = 0.087 (in Fig. 3d, the background of image is removed by setting threshold at 3 times of noise level).

DISCUSSION: As illustrated in Fig. 2, the reconstruction is stable because it is over-determined in every iteration step and each inner optimal solution is always close to the measurement in a least mean square sense. And it is efficient as no Fourier transform used in the inner loop. Fourier transform is expensive in terms of computation. The reconstruction is accurate, as illustrated in Fig. 3. It is important to know that, in the convex optimization, any locally optimal value is also globally optimal. Hence, the solution of inner optimization should converge to the global sparse solution. The L1 norm may slightly “shrink” the recovered signal and enlarge the difference (error in Fig. 3d).