High fidelity imaging using frequency sweep encoding

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Introduction: Recently, Frydman et al. proposed a novel mechanism for directly forming images in k space using frequency sweep encoding (1, 2). It relies on the quadratic dependence of magnetization phase on position. In combination with EPI-type readout, this method has found applications in single-shot spin-echo imaging of human and rat brains (3, 4). Its sequential excitation of magnetization may also be useful for novel image contrast generation and spectroscopic imaging. Fidelity of the images directly formed in k space is, however, severely degraded when compared with images formed using conventional phase encoding schemes with Fourier transform reconstruction. We show that fidelity of this type of images can be restored and also extend frequency sweep encoding to susceptibility-weighted imaging.

Theory: Using a RF pulse with linear frequency sweep such as a Chirp pulse and a simultaneous encoding x gradient, magnetization is sequentially excited along the direction of the encoding gradient. The phase of the magnetization at the end of sequential excitation has a quadratic dependence on the x position of the magnetization:

\[ \phi_{\text{readout}}(x) = ax^2 + bx + c \]  

where the constants a, b, and c depend on frequency sweep and gradient parameters (1, 2). Upon reversing the polarity of the encoding gradient an echo is formed in k space with the following phase relationship:

\[ \phi_{\text{readout}}(k(t), x) = \phi_{\text{readout}}(x) - \pi k(t) x \]  

where k(t) represent k values as a linear function of time. Using Frydman’s argument, signals at a certain k(t) point arise mostly from spin packets at x, with \( x_k \) determined by \( |d\phi_{\text{readout}}(k(t), x)| \approx 0 \). At \( x \neq x_k \) signals are largely dephased by the quadratic phase profile around \( x_k \). As a result, an image can be directly formed in k space with \( S(t) = -\gamma(x_k) \). Unlike the sinc point spread function associated with conventional Fourier imaging, signal dephasing in k space based on the quadratic phase profile (Eq. [2]) is incomplete. For the same number of pixels, the actual image resolution in the frequency sweep encoding direction is therefore significantly degraded, accompanied by significant geometrical distortions. In the reconstruction scheme proposed here the k space signal S(t) can be expressed in matrix form:

\[ S = FD \]  

where S, F and D are matrices for k space signal (M × 1), Frydman encoding (M×N), and relaxation-weighted spin density (N×1), respectively, with M ≥ N. The elements of matrix F are given by a phasor \( s_{mn} = \exp(i\phi_{\text{readout}}(k(t), x) \cdot x_k) \), where \( \phi' \) is an additional phase term to improve the conditioning of the Frydman encoding matrix F. Eq. [3] can be solved using least square:

\[ D = (FF^†)^{-1}FS \]  

where \( F^† \) denotes conjugate transpose. For M = N, Eq.[3] can be factored into:

\[ S = F^†\Phi D \]  

where \( \Phi \) is a diagonal matrix with element \( \phi_{mn} = \exp(i2\pi n FOV/N^2) \) and \( F^† \) is the conventional Fourier matrix. Eq. [5] allows direct Fourier transform of the k space image to generate an image in the x space with a quadratic phase profile given by \( \Phi \).

Methods and Results: Experiments were performed on a Bruker 11.7 Tesla scanner. SD rats (~200g) anesthetized using a mixture of 70% N2O, 30% O2, and 1.5% isoflurane were imaged using a 1-cm diameter surface coil for transmit and receive. The 2D version of the imaging sequence was shown in Fig. 1. A 8-ns chirp pulse of \( \theta \) (nominal 90) degree with both edges rounded off was used for Frydman’s frequency sweep encoding. A 2-mm slice is selected using one 180° MAO pulse or two identical sech pulses. For 3D imaging, the 180° slice selective pulse(s) and slice gradient are replaced by phase encoding along the third dimension. To minimize geometrical distortion, the k space image was read out to allow equal TE for the sequentially excited magnetization (3, 4). FOV = 20 mm x 20 mm. Acquisition time = 8 ms. Matrix size = 512 x 512. TR = 1 s. No. of averages = 4. T2* delay time = 10 ms. For Fourier reconstruction, the k space data were directly Fourier transformed. A quadratic phase profile calculated from Bloch simulation of the frequency sweep pulse was used to correct the 2nd order phase of the Fourier transformed image along the Frydman encoding dimension. After 2nd order phase correction, the image data were inverse Fourier transformed in both dimensions. The central 32 x 32 k space data were apodized using a 2D Hanning filter and then Fourier transformed to generate a low-pass filtered 512 x 512 image. Finally, the original 2nd order phase corrected image was demodulated using the phase of the low-pass filtered image (5). The results are shown in Fig. 2, top row. Using the same data, the images formed using only 1D Fourier transform along the conventional phase encoding direction are shown in Fig. 2 bottom row. The same 2nd order phase correction was applied. Clearly, geometrical distortion and blurring along the frequency sweep encoding direction can be minimized using the procedure described in Theory (results using Eq. [4] are not shown).

Conclusions: Combining Fourier transform and frequency sweep encoding allows formation of high fidelity images comparable to conventional images, thus giving impetus to develop new imaging methods utilizing Frydman’s sequential excitation and encoding with continuous frequency sweep.