A Hybrid L0-L1 Minimization Algorithm for Compressed Sensing MRI

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INTRODUCTION

Both L1 minimization [1] and homotopic L0 minimization [2] techniques have shown success in compressed-sensing MRI reconstruction using reduced k-space data. L1 minimization algorithm is known to usually shrink the magnitude of reconstructions especially for larger coefficients [1, 3] and non-convex penalty used in homotopic L0 minimization is advocated to replace L1 penalty [3]. However, homotopic L0 minimization only finds local minimum which may not be sufficiently robust when the signal is not strictly sparse but also has small elements after a sparsifying transform or the measurements are contaminated by noise [4]. Since practical MR images are never strictly sparse after a transform, it is desirable to estimate both large and small coefficients more accurately. In this abstract, we propose a homotopic L0-L1 hybrid minimization algorithm to combine the benefits of both L0 and homotopic L0 minimization algorithms for MRI. The proposed algorithm minimizes the L0 quasi-norm of large transform coefficients but the L1 norm of small transform coefficients for the image to be reconstructed. The experimental results show the proposed algorithm outperforms either homotopic L0 or L1 minimization when the same reduction factor is used.

THEORY AND METHOD

The proposed algorithm is formulated as solving the following optimization problem: \( \min_x \|x\|_0 \) s.t. \( y = \Phi x \), where \( \|x\|_0 = \sum_i f(x_i) \) is a hybrid L0-L1 quasi-norm with \( f(x_i) = \begin{cases} |x_i| & |x_i| < \tau \\ 1 & |x_i| \geq \tau \end{cases} \), and \( \tau \) is the threshold between the choice of L1 norm for small elements and L0 quasi-norm for large elements. Similar to the homotopic L0 minimization algorithms, the desired minimization problem in (1) is approximately solved by a sequence of L1 minimization problems minimizing \( \sum_i \rho(x_i, \epsilon) \). In this case, the function \( \rho(x_i, \epsilon) \) is chosen to be concave and approach the desired hybrid L0-L1 quasi-norm function as a sequence limit: \( \lim_{\epsilon \to 0} \rho(x_i, \epsilon) = \frac{a|x_i|/\tau}{|x_i|/\tau - b + \epsilon} \) for measuring noise suppression and edge preservation [7] were calculated, which take larger values when \( |x_i| \leq \tau \). Figure 1 shows how the function \( \rho(x_i, \epsilon) \) approaches the desired hybrid quasi-norm when \( \epsilon \) approaches zero for \( \tau = 1 \). In our choice for the cost function, strict concavity is the key to assuring solution uniqueness for the compressed sensing reconstruction problem [2]. Although a similar hybrid L0-L1 quasi-norm function has been applied to the finite difference of an image as a prior for SENSE regularization [5], the proposed hybrid quasi-norm is used in the context of compressed sensing, where random undersampling is used to construct an underdetermined linear equation and the hybrid quasi-norm is applied to the coefficients of any sparse transforms. Simulation was carried out to compare the proposed algorithm with L1 and homotopic L0 minimizations. Iteratively reweighted L1 minimization [6] was used among existing homotopic L0 minimization algorithms. An undersampled radial trajectory was used to generate the simulated k-space data and Gaussian noise was then added on the data. Two parameters \( a, b \) for measuring noise suppression and edge preservation [7] were calculated, which take larger values when image quality improves.

RESULTS AND DISCUSSION

Figure 2 shows the plot of sorted magnitude of a 256 × 256 phantom. It shows the phantom consists of some large elements which decay rapidly and many small elements. Figure 3 shows the reconstruction results for the phantom simulation. The reduction factor is 3.3 and the SNR is 20dB. Identity transform was used as the sparsifying transform. Parameter \( \tau \) was chosen to be 0.55 which is the “corner” of the plot in Figure 2. The algorithm and corresponding \( a/b \) (%) are labeled on the top and bottom of each reconstructed image. In addition, the corresponding “comb” region in the phantom was zoomed to reveal details. It is seen homotopic L0-L1 hybrid minimization algorithm outperforms the other two algorithms both visually and in term of \( a/b \) values. It suggests the proposed algorithm better preserves details and suppresses noise and artifacts. Future work will investigate optimal choice of parameters \( \tau \) in absence of prior knowledge.

CONCLUSION

A novel algorithm is proposed to integrate L1 and homotopic L0 minimizations for compressed-sensing MRI reconstruction. The results show that the proposed algorithm can outperform the L1 and hybrid homotopic L0 minimization algorithms in preserving details and suppressing noise and artifacts.

REFERENCES
