Total Generalized Variation (TGV) for MRI

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Introduction: Total Variation (TV) based strategies, which were originally designed for denoising of images [1], have recently gained wide interest for many MRI applications such as regularization in image reconstruction and deconvolution. TV models have the main benefit that they are very well suited to remove random fluctuations and noise-like artifacts from sub-sampled scans (radial or random), while preserving the edges in the image. However, the assumption of TV is that the images consist of regions which are piecewise constant. Inhomogeneities of the coil sensitivities and the exiting B1 field often violate this assumption in practical MRI examinations. Additionally, there are also natural gradually changing signal intensities in the investigated anatomies. In all these situations TV leads to staircasing artifacts and results in patchy images, which appear unnatural. This paper introduces the new concept of Total Generalized Variation (TGV) as a penalty term for MRI problems. This mathematical theory has recently been developed [2], and while it is equivalent to TV in terms of edge preservation and noise removal, it can also be applied in imaging situations where the assumption that the image is piecewise constant is not valid. As a result, the application of TV in MR imaging is far less efficient, TGV results show a less blocky, more natural appearance. This paper introduces the new concept of Total Generalized Variation (TGV) as a penalty term for MRI examinations. Additionally, there are also naturally gradually changing signal intensities in the investigated anatomies. In all these situations TV leads to staircasing artifacts and results in patchy images, which appear unnatural. This paper introduces the new concept of Total Generalized Variation (TGV) as a penalty term for MRI problems.

Discussion: This work presents TGV for MR imaging, a new mathematical framework which is an extension of the TV method. While it shares the existing desirable features of TV, it has additional benefits, which allow using TGV in MR imaging situations where TV fails because of its model assumptions being violated. Additionally, due to the absence of staircasing artifacts in TGV, even in the case of homogeneous images, it delivers images that follow much better the underlying signal changes. It is possible to use TGV in all applications where TV is currently applied. The denoising and image reconstruction experiments of this work showed clearly improved image quality over conventional TV. TGV is based on a solid mathematical theory within the framework of convex optimization, which ensures that numerical implementations are usually straightforward as a variety of numerical algorithms already exist for these types of problems. Concerning additional applications, it is worth noting that it is possible to use TGV as a regularizer that can be applied directly to individual channel array coil data. Thus, it is possible to use TGV in combination with all parallel imaging reconstruction methods that first generate individual coil images, which are then combined to a single sum of squares image in the last step, like GRAPPA. Another promising application for TGV would be diffusion tensor imaging. Currently, TGV is only defined for scalar functions, but as symmetric tensor fields are inherent in the definition, it can be easily extended to higher order tensor fields and could therefore be used as a regularizer to reconstruct the diffusion tensor.

Methods: The notion of k-th order TGV bases on taking the relevant information from the first k derivatives of an image into account. It is a generalization in the sense that TGV1 is equivalent to TV which only incorporates first-order derivative information. In this work, only TGV2 is discussed and it is shown that TGV; constitutes a suitable regularization for piecewise smooth MR images. One of its main features is the absence of the staircasing effect for data violating the piecewise constance condition. Mathematical details of the functional and proofs can be found in [2], but an intuitive explanation of the reason for this is as follows:

Formally, it holds that \[ TGV_2(u) = \inf_a \left\{ \int \left( |\nabla u| + \frac{1}{2} |\nabla^2 u| \right) dx \right\} \] where \( \nabla \) is the gradient operator and \( \nabla^2 \) the Laplacian. In total, natural looking piecewise smooth images are penalized less than staircase images and therefore preferred in TGV2-based minimization problems. This is shown in Fig. 1, which illustrates the absence of staircasing artifacts in TGV2 in contrast to conventional TV for denoising of a numerical ramp image.

The application of TGV for image denoising is of the form \[ \min_{u \in H^2} \int \left( |\nabla u| + \frac{1}{2} |\nabla^2 u| \right) dx \] for a noisy image \( f \). A contrast enhanced T1 weighted clinical measurement of the prostate was performed with a 3D gradient echo sequence. All measurements were performed on a clinical 3T system (Siemens TIM Trio, Erlangen, Germany) and written informed consent was obtained from all subjects prior to the examinations. Sequence parameters were TR=TE=3.1ms, FA=15°, matrix size 256x256, 20 slices, slice thickness 4mm, in plane resolution 0.85x0.85mm. The measurements showed severe signal inhomogeneities due to the exiting b1 field [4] which violated the assumption that the images consist of piecewise constant areas.

For iterative image reconstruction of undersampled radial data from multiple coils, we extended the approach proposed in [3] with an integration of a TGV2 constraint. Undersampled T2 weighted radial spin echo measurements of the human brain were performed using a receive only 12 channel head coil. Sequence parameters were: TR/TE=2500/50ms, matrix size 256x256, slice thickness 2mm, in plane resolution 0.78x0.78mm.

Results: Fig. 2 shows the results from the denoising experiments. While TV and TGV2 have the same denoising behaviour with sharp edges remaining in the image, TV results show significant staircasing artifacts in regions where inhomogeneities violate the assumption of piecewise constant images. Figs. 3 and 4 compare the reconstructions from undersampled radial imaging. Reconstructions from 48, 32 and 24 projections are displayed. It is not surprising that conventional NUFFT reconstructions show streaking artifacts which get worse as the number of projections is reduced. While both TV and TGV2 reconstructions eliminate streaking artifacts efficiently, TGV2 results show a less blocky, more natural appearance.

Fig. 1: Illustration of TV (middle) and TGV2 (right) denoising of a numerical ramp image.

Fig. 2: Image denoising: Original image (left), TV (middle) and TGV2 (right). The bottom row shows magnified views from a region with severe signal inhomogeneities.

Fig. 3: TGV2 Image reconstruction of undersampled radial data with 48, 32 and 24 spokes. Conventional NUFFT reconstruction (left), TV (middle) and TGV2 (right).

Fig. 4: Magnified views from Fig. 3. Conventional NUFFT reconstruction (left), TV (middle) and TGV2 (right).
