Thickness profile generation for the corpus callosum using Laplace's equation

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Introduction

Analysing the thickness profiles of the corpus callosum (CC) in MR images has been important in showing morphological abnormalities in numerous psychiatric and neurological disorders. We present a novel method, based on the Laplace equation technique (1) to measure cortical thickness, for measuring the thickness profile of the CC in mid-sagittal slices of MR images. The method improves on previous work (2) since it guarantees that thickness measurements are derived from non-intersecting, cross-sectional traversals of the callosum. We compare the results of statistical comparison of callosal thickness profiles of the new method, the \textit{Laplace equation thickness} method, with results produced by previous work, the \textit{straight line callosal thickness} method (2), on a dataset encompassing various stages of schizophrenia.

Methods

Participants: We recruited 86 patients with established or chronic schizophrenic illness (CSZ), 110 individuals with first-episode psychosis (FEP), 100 individuals at ultra-high risk for psychosis (UHR; 27 of whom later developed psychosis, UHR-P, and 73 who did not, UHR-NP), and 55 control subjects (CTL).

MR imaging: All subjects were scanned on a 1.5T GE Signa MRI machine. A three-dimensional volumetric spoiled gradient recalled echo in the steady state sequence generated 124 contiguous, 1.5mm coronal slices. Imaging parameters were: TE, 3.3msec; TR, 13.4msec; flip angle, 30\textdegree; matrix size, 256 × 256; FOV, 24 ×24cm matrix; voxel dimensions, 0.938 × 0.938 × 1.5mm.

Image segmentation: We utilised a semi-automated segmentation pipeline (2), which employed automated skull stripping, registration, and thresholding methods followed by manual corrections. The rostral and caudal endpoints were marked (see Figure 1(i)) in order to divide the callosum into superior and inferior contours.

Measuring callosal thickness: Utilising the formulation in (1), we model the callosal thickness, for a point, \( x_0 \), inside the callosum, as the length of the streamline of Laplace’s equation that traverses the callosum from the superior to the inferior boundary and passes through \( x_0 \). Formally, Laplace’s equation is a second order partial differential equation that defines the scalar-valued potential field, \( \phi \), defined for each voxel within the CC, that is enclosed by the superior and inferior boundaries of the callosum. We employ the Dirichlet problem form of Laplace’s equation which is defined, in two dimensions, as,

\[
\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \quad \left\{ \begin{array}{l}
\phi_{\text{superior}} = \text{constant} \\
\phi_{\text{inferior}} = \text{constant}
\end{array} \right. \quad \phi_{\text{uperior}} \not= \phi_{\text{inferior}}, \quad \phi_{\text{superior}} \in \mathbb{R}, \quad \phi_{\text{inferior}} \in \mathbb{R}.
\]

For convenience, we chose \( \phi_{\text{uperior}} = 0 \) and \( \phi_{\text{inferior}} = 1 \). The Finite Difference Method with the standard five-point template was employed to discretise equation [1] into a symmetric, positive-definite system of linear equations. The iterative Conjugate Gradients method was used to solve for \( \phi \). The streamlines were initialised by subdividing the equipotential contour \( \phi = 0.5 \) into 41 equally spaced nodes, of which the extreme 2 points were discarded (see Figure 1(i)). The standard first-order Euler integration technique was used to construct the streamlines, initialised at points previously denoted \( x_0 \), based on the normalised gradient of \( \phi \). The arc length of each streamline is the \textit{Laplace method callosal thickness}. The mathematical properties of Laplace’s equation guarantee that the contours are non-intersecting, nominally parallel and intersect both boundaries orthogonally.

Results

A comparison of group-wise statistical analysis results generated by the straight-line and Laplace methods can be seen in Figure 2, with significant regional between-group changes mapped to a mean callosum. When CSZ were compared to CTL, a similar profile of significant reductions was seen in the anterior and posterior genu, and in the isthmus (p<0.0001 using the Laplace method, p<0.0001 using the straight line method). When the FEP and CTL groups were compared, no significant findings were seen in the Laplace method (p=0.116), whereas changes were seen in the genu using the straight-line method (p<0.01). When UHR and CTL groups were compared, a trend was seen in both methods (p=0.092 using the Laplace method, p=0.079 using the straight-line method). When the UHR-P and UHR-NP groups were compared, a trend was seen in both methods (p=0.092 using the Laplace method, p=0.079 using the straight-line method). When the UHR-P and UHR-NP groups were compared, a trend was seen in both methods (p=0.092 using the Laplace method, p=0.079 using the straight-line method).

Conclusions

We utilised an adaptation of the method in (1) for measuring thickness between the superior and inferior surfaces of the corpus callosum. This produced similar, albeit arguably more conservative, results in between-group comparisons than a straight-line method across a large dataset in whom robust and meaningful differences have previously been demonstrated. The primary contribution of this work was a novel, robust, and computationally efficient method for measuring the thickness of the corpus callosum in MR images. The method demonstrably produced similar statistical analysis results as the straight line method.

References


![Figure 1: (i) Laplace equation thickness model shown on an idealised corpus callosum. A subset of the streamlines are annotated with their indices, which were chosen to increase rostrally.](image)

![Figure 2: Comparison of statistical analysis results using the straight line callosal thickness measure (first row) and the Laplace method callosal thickness measure (second row). The columns of the figure denote the CSZ vs. CTL (i), FEP vs. CTL (ii) and the UHR-P vs. UHR-NP (iii) contrasts.](image)