Quantitative Calculation of the Proton Radiation Damping Constant at 14.1 tesla

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Introduction: Detection in magnetic resonance is dissipative, inasmuch as the receiver presents a resistive load to the antenna; and the damping of the transient free induction signal in pulsed NMR, as the spins lose energy to the receiver, is called radiation damping. This was first analyzed, a half century ago (1), in the classic work of Bloembergen-Pound (BP), who formulated the damping constant in terms of two global figures of merit: the filling factor \( \eta \) – usually interpreted as the ratio of sample to coil volume --, and the quality factor \( Q \), which measures stored energy versus dissipation per cycle. Modern measurements on analytical systems at 12 tesla and above give the product of \( \eta Q \) in the range of 7 to 10, for protons (2). Since the probe \( Q \) factors in this case are typically in the range of 250 to 400, the dictates improbably small values for \( \eta \), in the range 0.03, suggesting the need for an alternate theory of the damping constant. It has elsewhere been shown shown (3) how to re-parameterize the damping constant in terms of the coil efficiency, designated \( \zeta \) – i.e. the peak \( B_1 \) in the laboratory frame, per unit current, divided by the coil resistance (4) – so as to eliminate both the \( Q \) and filling factors. We here report quantitative agreement of such calculations with measurements on protons at 14.1 tesla.

Experimental Methods: It has long been known (5) that radiation damping is sensitive to tuning of the NMR probe, and that its rate can be greatly reduced by severe de-tuning. We have therefore determined a lower bound on the damping constant as the difference in linewidth between spectra acquired with a tuned and a detuned probe. NMR measurements were done on a Varian AVOIA spectrometer at a static field strength of 14.1 tesla, giving a proton Larmor frequency of 600 MHz. The proton channel of a 5 mm \(^{13}\text{C}^{15}\text{N}\) inverse detection probe was used. Return loss at 600 MHz was measured at -30 dB with an Agilent 5071 network analyzer, and -1 dB for the probe detuned using its own tune and match capacitor wands. The sample was de-ionized H2O, with no added D2O, and all measurements consisted of single pulse and acquire sequence, run with sample spinning and no field-frequency lock. Small tip angles were employed, in the range of 0.1 to 1 degree. To determine the efficiency, the \( B_1 \) was measured using the spectrometer software to seek a nutation of \( 2 \pi \) after calibrating all cable losses with the network analyzer. The calculated efficiency \( \zeta \) was corrected for the fact that we measure the peak \( B_1 \), but rms power. In fact, a nutation of \( \pi/2 \) was obtained with a square pulse of 8.5 \( \mu \)s duration, at 16 W of power absorbed by the probe.

Results and Discussion: Figure (1A) shows the free induction decays with the probe detuned (above) and tuned (below). The effect of damping is strongly marked in the latter. Figure (1B) shows the corresponding spectroscopic lines, superposed on the same axis, and normalized to equal peak-height.

The linewidths (full width at half height) as determined by the spectrometer software, are 45.0 Hz and 1.0 Hz, giving 44 Hz as a lower bound for the damping linewidth. Aside from detuning the probe, and a slight adjustment in pulsewidth, the spectrometer conditions are identical for both measurements, which were made minutes apart. Given that the theoretical damping lineshape is Lorentzian for very small tip angles (6), a theoretical linewidth of of \( k/\pi = 46.6 \) Hz was predicted, from a calculated damping constant \( k = 146.4 \). Here \( k = \gamma \omega M_0 \zeta^2 / 4 \), \( V \) is the sample volume, \( M_0 \) is the equilibrium magnetization at 298 K, and other symbols have their usual meanings. While the close agreement to the experimental number may be judged fortuitous at this point, it is clear that calculation of the damping rate based upon the efficiency is superior to that using the filling factor. In fact, using the measured \( Q \) of the probe, and assuming a filling factor of 1, we calculate, using the BP formula in SI units (7), a linewidth of 338 Hz, which far exceeds the measured value.

References:
5. A. Abragam, Principles of Nuclear Magnetism, Oxford (1961), Ch. II.