Non-central Chi model in GRAPPA: experimental results

Here we present an experiment supporting this claim. Assuming a simplified model in which the variance of noise in the k-space is the same in all the coils, and no correlation exists, then Eq. 1 can be reduced to give the power of noise in each coil as

\[ \sigma_k^2(x) = \sum_{l=1}^{N} \langle W_k(x) \rangle^2 \sigma_l^2 \]

where the variance of noise in the k-space is originally the same in all the coils, and no correlation exists between them. We have found that in practice, this is not the case; the non-central Chi model does not hold in general for GRAPPA reconstructions. However, under certain conditions (which we present below), the variance of noise is nearly constant for all positions and coils, so a non-central Chi model may adequately fit the data, like the Rician model does for conventional MRI.

Fig. 1: CV between coils of noise power \( \sigma_k^2 \) for different acceleration rates from an 8 coils acquisition, GE Sigma 1.5T EXCITE scanner, FSE Pulse Sequence, TR=500 ms, TE= 13.8 ms, matrix size= 256x256, FOV=20x20 cm, slick thickness= 5 mm. 15 ACS lines are considered for weights determination.

Non-central Chi model in GRAPPA: statistical analysis

The noise distribution in each coil after GRAPPA reconstruction is known to be Gaussian and non-stationary (i.e. spatially dependent), and correlation exists between coils. As a consequence, the variance of noise in each coil will be Gaussian but non-stationary. If the reconstruction is made using Sum of Squares (SoS) [Const97] the composite magnitude image will follow a non-central Chi distribution only if the variance of noise in one pixel is the same for all coils, and no correlation exists between them. We have found that in practice, this is not the case; the non-central Chi model does not hold in general for GRAPPA reconstructions. However, under certain conditions (which we present below), the variance of noise is nearly constant for all positions and coils, so a non-central Chi model may adequately fit the data, like the Rician model does for conventional MRI.

Fig. 2: Relative errors in the PDF for the non-central Chi squared approximation, as a function of the CV of the noise power at each coil. Curves are parametrized depending on the SNR. Systems with 8 (left) or 12 (right) receiving coils are considered.