Adiabatic pulses revisited through averaging

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Introduction: The use of adiabatic passages is popular in magnetic resonance as a result of their robustness to the RF field inhomogeneities [1, 2], thus a computationally efficient method of solving the Bloch equation for adiabatic pulses is important for design purposes. Here, the Bloch equation is scaled and subsequently averaged to find the magnetization behaviour in a straightforward way with negligible error for adiabatic passages. The novel framework presented here may be used to optimise the adiabatic modulation functions [1, 3].

Theoretical Results: Radio frequency waveforms in MR systems are generated by piece-wise constant signals (Fig. 1), and pulse design is based on the Bloch equation. By neglecting the relaxation time constants during the excitation period, the Bloch equation in the classical rotating frame of reference is \[ \sigma' = \sigma(t) \sigma(x', y', z') \] in which \( \sigma_x = (\cos(\omega t) - \sin(\omega t)) \). We have studied the behaviour of adiabatic passages through a first order averaging technique used the in nonlinear dynamical systems theory [4]. A surprising result is that in this novel representation of the Bloch equation, the solution is given by a single matrix exponential, and is therefore an extremely computationally efficient method. Simulation results demonstrate the negligible error that can be proven analytically. The method can be directly applied to aid the design of adiabatic passages in MRI.

Simulation Results: Fig. 2 shows the simulation results for a typical Adiabatic Full Passage (AFP) chirp signal with amplitude of 117 µT and frequency sweep of \( 2 \times 10^4 (1-800t) \) Hz as in [1]. As shown in this figure magnetisation behaviour from the scaled Bloch equation is a slowly varying signal compared to the magnetisation in the classical rotating frame of reference. The error at the end of the pulse excitation period is less than three percent (Fig 2.d).

Conclusions: We have studied the behaviour of adiabatic passages through a first order averaging technique used the in nonlinear dynamical systems theory [4]. A surprising result is that in this novel representation of the Bloch equation, the solution is given by a single matrix exponential, and is therefore an extremely computationally efficient method. Simulation results demonstrate the negligible error that can be proven analytically. The method can be directly applied to aid the design of adiabatic passages in MRI.