Optimal Multi-echo Water-Fat Separated Imaging Parameters for Temperature Change Measurement using Cramer-Rao Bounds

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Introduction

MR thermometry of tissue containing mixtures of fat and water using proton resonant frequency shift (PRFS) techniques is difficult due to the lack of PRFS shift in the fat signal. Multi-echo fat-water fitting techniques that separate the fat and water effects have been shown to be useful in measuring temperature in fat-water phantoms[1]. In this study we explore optimization of echo time selection by minimizing the temperature noise using Cramer-Rao Lower Bound (CRLB) analysis. Accuracy of fitting is improved by including multiple fat peaks and T2* effects. Our approach finds the minimum temperature noise that has the minimum sensitivity to the values of nominally fixed parameters.

Methods

The CRLB was derived for a model of MR signal with contributions from both water (1 peak) and fat (3 peaks), given in equation 1.

\[
s(n) = \left( A_w e^{-\frac{TE(n)}{T2_w}} e^{2\pi i \nu \Delta T} + A_f e^{-\frac{TE(n)}{T2_f}} \left( \frac{\phi_{fw1}}{\beta_1} e^{2\pi i \nu (n-1) T2_f} + \frac{\phi_{fw2}}{\beta_2} e^{2\pi i \nu (n-2) T2_f} + \frac{\phi_{fw3}}{\beta_3} e^{2\pi i \nu n T2_f} \right) \right) e^{2\pi i \nu \Delta T} + \epsilon
\]

where \(A_w\) and \(A_f\) are the TE = 0 amplitudes at the TR of interest for water and fat signal contributions. \(T2^*_w\) and \(T2^*_f\) are the T2* values for water and fat, \(\alpha\) is the PRFS thermal coefficient (0.01ppm/°C), \(\nu\) is the imaging frequency, \(\phi_{fw}\) are the frequency differences between the fat and water peaks at a “baseline” temperature \(T_b\), \(\psi\) is the signal offset due to B0 frequency shift, and \(\epsilon\) is Gaussian noise with mean=0 and variance=\(\sigma_\epsilon^2\). \(\beta_i\) is the relative ratio of the area of each fat peak compared to the area of all fat peaks combined, with summed values adding up to 1. \(\Delta T\) is the temperature change from base temperature \(T_b\) and \(n\) is the echo number.

Echo time sets were the independent variable and were parameterized with three variables, all related to the phase angle between the water and fat signals as: \(TE(n) = \frac{\phi_{fw1}}{\beta_1} + \frac{\phi_{fw2}}{\beta_2} + \frac{\phi_{fw3}}{\beta_3}\). Using methods described by Pineda, et al. [2], the CRLB was calculated for \(\Delta T\) for a given echo-time set (\(n\), rotation, starting value and separation angle) for a wide range of the parameter values (\(\Delta T, T2^*_w, T2^*_f, A_f/A_w, \phi_{fw1}, \phi_{fw2}, \phi_{fw3}\)). For all calculations, SNR = 1, \(\psi=-12.5\)Hz, \(\phi_{fw1} - \phi_{fw2} = -47\)Hz, \(\phi_{fw1} - \phi_{fw3} = -255\)Hz, \(\beta_1 = 0.82\), \(\beta_2 = 0.11\), and \(\beta_3 = 0.07\). The standard deviation of the CRLB values of \(\Delta T\) (for each echo-time set) was calculated giving an overall noise value for that set of \(TE(n)\). To confirm these estimates of the noise in \(\Delta T\) an experiment was performed using fat-water phantoms with 70:30, 50:50, and 30:70 fat:water content. The phantom was imaged at 1.5T with several starting/separation angle pairs (both “good” and “bad” choices) using \(k=3, 5, \) and 7. The images were processed with a nonlinear fitting algorithm that fits equation 1. The standard deviation of the temperature values in a 1000 pixel ROI was calculated (constant temperature). These values were then compared to CRLB values calculated using fitted values of \(A_w, A_f, T2^*_w, T2^*_f, \beta_1, \beta_2, \) and \(\beta_3\).

Results

It was found that rotation=4 gave the lowest CRLB noise and variation. Figure 1 shows a plot of the standard deviation of the temperature CRLB when \(\Delta T, A_f/A_w, \) and fat-water frequency difference are varied. The white regions indicate large values of standard deviation. Figure 2 shows the results of the phantom experiment for the 50:50 phantom and 5 echoes.

Discussion

As seen from the minimum region in Figure 1, the most robust starting and separation angle pair is in the dark regions of the plot approximately equal to rotation=4, starting angle = 215° and separation angle = 240°. As seen from Figure 2, our CRLB estimates closely match the temperature noise values found in the 50:50 phantom. Other phantom results are similarly matched.

References: