Fast optimization method for general surface gradient coil design

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Introduction
Over the past 20 years many papers have discussed theoretical design methodologies for MRI gradient coils. She et al. [1] presented a method based on finite element method to design a gradient coil. Compare with the standard target field method, this is an attractive method because the design procedure can be employed to also consider the eddy current effects and an inhomogeneous analysis domain. However, the very large computational efforts induced by finite element analysis limit the practical application of this method.

In this paper, we present an efficient numerical iterative optimization method for designing linear gradient coil on a current-carrying surface. Using the scalar stream function as design variable, the value of the magnetic field inside a computational domain is calculated using the least square finite element method. The first-order sensitivity is calculated using the adjoint equation method. The detailed numerical optimization skills are discussed in order to obtain a fast and effective optimization procedure.

Theory
The optimization objective used in this paper is the least square type function

$$F = \int_{\Omega_{ROI}} \frac{\partial B}{\partial x} - G_i \, d\Omega,$$

where $G_i$ is the linear x-gradient of specified magnetic field, $\Omega_{ROI}$ is the region of interest (ROI) and the magnetic field $B_i$ is calculated using the Least-squares finite element method [2]. That is to find $B_i = (B_{i1}, B_{i2}, B_{i3})^T \in \mathbb{V}_h$ such that $\alpha(B_i, \nabla \psi) = f(\psi)$ for all $\psi \in \mathbb{V}_h$, where $\alpha(B_i, \nabla \psi) = \int_{\Gamma_{coil}} (\nabla \times \nabla \psi) \cdot \nabla \psi \, d\Gamma + \int_{\Omega_{ROI}} h^T \left( \nabla \times \nabla \psi \right) \cdot \nabla \psi \, d\Omega, \psi = 1$.

Here, $\psi$ is the Lagrange finite element space of computational domain $\Omega = \Omega_{ROI} \cup \Omega_{coil}$ (ROI is inside of $\Omega_{coil}$), $h$ is the size of mesh, $\mathbb{V}_h$ is the unit normal vector on the boundary $\Gamma$ or current-carrying surface $\Gamma_{coil}$ (figure 1), \[u(x) := \lim_{\delta \rightarrow 0} u(x + \delta) - u(x - \delta)\] with $x \in \Gamma_{coil}$ denotes the jump of $u$ across the $\Gamma_{coil}$, $\mu$ is the permeability and the surface current density $j$ can be expressed as $j = \nabla \times (\psi \mu)$ using the stream function $\psi$ which is the design variables.

The design of gradient coil is an inverse problem. One needs to use the regularization technique to avoid the oscillation of the coil layout. Typically the inductance of the coil, or the magnetic energy term is combined to obtain a reasonable layout of the coil. In our example, we use the limited-memory BFGS method [3] with filter technique to implement the regularization effect. In the limited-memory BFGS, the first-order sensitivity of our objective can be obtained by the following formula $\frac{\partial L}{\partial \psi} = \frac{\partial F}{\partial \psi} + \alpha \left( \frac{\partial K}{\partial \psi} \right)$. Here $L = F(\psi) + \alpha (J-K)$ is the corresponding discretized Lagrangian model of the original optimization problem, $F$ is the discretized expression of objective function in equation (1), $K$ is the Lagrange multiplier, $J$ is the discretized vector of surface current density and $\mathbb{V}_h$ is the discretized global stiffness matrix using the LSFM. The Lagrange multiplier $\alpha$ can be obtained by solving the following adjoint equation $K^T \alpha = -\frac{\partial F}{\partial \psi}$.

An iterative optimization of gradient coil design includes several key steps which are run sequentially (figure 2). The main computational cost for each iteration is the steps of the finite element analysis and sensitivity analysis. When the LSFM is used to discretize a design domain, the discretized stiffness matrix $K$ is kept unchanged and merely needs to be assembled once for all iterative steps. Therefore only the vector on the right hand side of the discretized equilibrium equation needs to be updated in each iteration. Based on this condition, one can perform the decomposition of the matrix $K$ merely once and save the decomposed matrices at the beginning of the optimization. Then only back substitution is performed to obtain the solution of LSFM and sensitivity. Because the time used for substitution step is much shorter than that used for the decomposition step (table 1), this strategy can be employed to speed up the whole optimization procedure.

Numerical results
Figure 3 shows an example of a Gx gradient coil on a cylindrical surface $\Gamma_{coil}$ with radius=45mm and height=270mm. The ROI is a cylinder with radius=30mm and height=42mm. The whole domain is discretized into a hexahedral mesh with 89610 points. The Gx gradient coil layout (contour lines of the stream function) on the $\Gamma_{coil}$ is shown in figure 3a. The gradient strength at the center of ROI is 10mT/m. Figure 4 shows layouts of two multi-layer Gx gradient coils for target gradient strength =10mT/m.

Discussion
This abstract presents a fast optimization procedure to design a surface gradient coil for MRI. Based on LSFM, numerical examples demonstrate that this method can be used to design a gradient coil on any surface.

Reference
3 J. Nocedal, S. J. Wright, Numerical Optimization, 1999

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