The intrinsic magnetic field symmetries of the spiral birdcage coil

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INTRODUCTION Transmit array spatial encoding (TRASE) is a novel MRI technique that utilizes $B_1$-field phase gradients as the means for traversing k-space [1-3]. This method allows MRI to be performed without the usual application of $B_0$-field amplitude gradients, thereby possibly eliminating the equipment, complexity, power consumption and acoustic noise associated with traditional static-field gradient coils. As a result, TRASE might provide a comfortable (i.e. silent) imaging experience in a lower-cost, simplified scanner.

The primary requirement of TRASE is an RF phase-gradient coil, which ideally generates a transverse RF field of constant magnitude and linearly varying phase over the sample volume. Previous work [1-3] did not focus on the design and optimization of RF phase-gradient coils. Rather, basic coil structures with roughly the correct field and phase symmetries were employed. Most notably, a spiral birdcage coil (Fig. 1) was used to generate a transverse RF field with phase varying along $B_0$. Here we explore the intrinsic properties of such a coil in order to guide and motivate the future design of phase-gradient coils.

APPROACH To begin, one must ask the question: does the ideal, transverse, phase-gradient field $B_1 = B_1(\cos(z)i + \sin(z)j)$ even satisfy Maxwell’s equations? Unlike its counterpart $B_1 = B_1i$ associated with conventional RF production by standard birdcage coils, the answer is no – at the very least, it turns out, a z-component must accompany the desired transverse field here.

To more thoroughly analyze the spiral birdcage coil, we use a Fourier transform method that was originally developed to design self-shielded gradient coils [4]. We previously used this technique to design active and passive shielding for contiguous, wire-wound, RF coils [5,6] for very low sensitivity) away from the isocentre. For a more sophisticated analysis, the axial component of the surface current of an actual spiral birdcage coil can be modeled as $F_z = F_1 \sin(\phi + \alpha z) \cos(\alpha \rho)$, where $h$ is the twist-per-unit-length of the coil and $\rho$ is the pitch angle of the spiral. Here the Fourier components of the axial surface current are $F_m = e^{im\phi}$, and its magnetic field inside the coil evaluates to

$$
\begin{align*}
B_\rho &= \mu_o F_1 a^2 \cos(\alpha) K_1(\rho a) \\
B_\phi &= h \rho \sin(\alpha z) I_1(\rho h) \\
B_z &= h^2 \sin(\alpha z) I_1(\rho h)
\end{align*}
$$

where $I_1$ and $K_1$ are modified Bessel functions (with prime denoting derivative) and $a$ is the coil radius. Here $B_\rho$ is non-zero and its magnitude depends on the radial position $\rho$. More importantly, the transverse field components are not uniform for a given $z$, but also vary as a function of $\rho$. Near the central axis, as $\rho \to 0$, the variation in these field components scales as $\rho^2$. The increase in magnitude of the transverse field (and hence also sensitivity) away from $\rho=0$ is consistent with the experimental observation that spiral birdcage coils exhibit less central image brightening [7].

For a more sophisticated analysis, the axial component of the surface current of an actual spiral birdcage coil can be modeled as

$$
F_z = \frac{1}{a} \cos(\alpha) \left( -\frac{h^2}{\rho} \cos(\alpha z) I_1(\rho h) + \frac{h}{\rho} \sin(\alpha z) I_1(\rho h) + h^2 \sin(\alpha z) I_1(\rho h) \right)
$$

where $I_1$ is the boxcar function truncating the coil at half-lengths $\pi l$, $N$ is the number of spiral rungs, $I_j(t)$ is time dependent current in each rung and the delta function sets the azimuthal location of each rung at angle $\Phi_j(z)$. The Fourier transform of $F_z$ and a complete analytic formulation of the spiral birdcage field follow quickly from this. As above, we find that even near the coil isocentre there is a strong radial dependence in the transverse field.

CONCLUSION The spiral birdcage coil and its archetypal surface current distribution (the infinitely-long, twisted, sine-phi) have been analyzed using the Fourier transform method laid out in Refs. [4,5]. The results show that twisting leads to a strong, intrinsic radial dependence in all magnetic field components. This knowledge will be used to guide the optimization of the next generation of phase gradient coils used for TRASE. It may also provide the basis of a more direct correction scheme for the problem of central brightening that occurs in high field imaging [7].