A Robust Parametric Method for Bias Field Estimation and Segmentation of MR Images

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Introduction
A major difficulty in quantitative processing and analysis of magnetic resonance (MR) images is intensity variation due to effects such as B1 and B0 field inhomogeneities. These effects can be represented as a bias field. It is often a mandatory step to correct for this bias before applying other automatic algorithms to images. One of the most popular types of methods for bias field correction involves segmentation (see Wells [1], and Leemput [2]). A common drawback of these methods is that they are sensitive to the initialization of the parameters. In this work, we propose a novel parametric method for joint image segmentation and bias field estimation for MR images. The bias field is parameterized as a linear combination of smooth basis functions. Image segmentation and bias field estimation are performed by minimizing a cost function. A desirable advantage of the proposed method is its robustness to initialization, which thereby allows fully automatic applications.

Method
Our proposed method is based on the following model of image formation in MR: \( I(x) = b(x) + J(x) + n(x) \), where \( J(x) \) is the measured signal, \( b(x) \) is the bias field, \( n(x) \) is noise, at location \( x \). We assume that the true signal \( J \) is piecewise roughly constant, and the bias field \( b \) is smoothly varying. In this work, we model the bias field as a linear combination of \( d \) smooth functions \( s_1, \ldots, s_d \), i.e. \( b(x) = \sum_{j=1}^{d} w_{j} g_{j}(x) \), where \( w_1, \ldots, w_d \) are the combination coefficients to be determined. In our current implementation, Legendre polynomials are used as the basis functions. The true signal \( J \) is approximated by \( J(x) = \sum_{j=1}^{d} c_j m_j(x) \), where \( m_j(x) \) is the membership function for the \( j \)-th tissue, \( c_j \) is the mean of the intensities in the \( j \)-th tissue. Let \( M = (m_1, \ldots, m_d) \), \( C = (c_1, \ldots, c_d) \), and \( W = (w_1, \ldots, w_d) \). Then, image segmentation and bias field correction can be formulated as the following optimization problem of minimizing the cost function \( \| M \cdot C \cdot W \| = \int \left| I(x) - \left( \sum_{j=1}^{d} c_j m_j(x) \sum_{j=1}^{d} w_j g_j(x) \right) \right|^2 dx \), subject to \( 0 \leq M(x), W(x) \leq 1 \) and \( \sum_{j=1}^{d} m_j(x) = 1 \). Note that the above cost function \( F \) is convex in the variables \( M, C, \) and \( W \). Therefore, there is a unique minimum for each variable when the other variables are fixed. The initial means and memberships of each class for our method are randomly generated. The algorithm consists of following 5 steps: 1. Initialization; 2. Optimize \( C \) for fixed \( W \) and \( M \). 3. Optimize \( W \) for fixed \( C \) and \( M \). 4. Optimize \( M \). 5. If converge, stop computation, otherwise go to step 2.

Results
Fig. 1 shows results of our method for three 3T brain MR images (column 1). The estimated bias fields, the segmentation results, and the bias corrected images are shown in the second, third, and fourth columns, respectively. It can be observed that the inhomogeneity has been corrected and the segmentation results are satisfactory. Fig. 2 shows comparison of the proposed method with methods of Wells et al. and Leemput et al., some gray matter (GM) is incorrectly identified as white matter (WM). Similarly, for the case of the method of Leemput et al., the intensities of the WM in the lower left of the ventricle are relatively lower, which results in wrongly labeling this area as GM. By contrast, our method achieves satisfactory corrected images and segmentation, as shown in the bottom row. To quantitatively evaluate the methods, we tested them using 30 2D images from McGill brain data [3]. The methods by Wells et al. and Leemput et al. are quite sensitive to initialization, which can be seen from the results from 20 initializations of the means, with the variances and a prior probability initialized by good guess from prior knowledge. To evaluate the performance of segmentation algorithms and bias correction, we use the Jaccard similarity (JS) [4]. The closer the JS value to 1, the better the segmentation and bias correction. The JS values for WM and GM obtained by the three methods are shown in Fig. 3. The JS values of the Wells et al., the Leemput et al. and our method are plotted with red squares, green circles and blue diamonds respectively. It can be observed that JS of our method are higher than the other two methods. It is worth highlighting that our method achieves almost the same JS values obtained from 20 different initializations. This experiment demonstrates a desirable robustness of our method to the initializations.

Conclusion
We have presented a novel parametric method to estimate the bias field in MR images. The bias field is modeled as a linear combination of smooth basis functions. A desirable property of the proposed method is that it is robust to the initialization, which allows fully automatic applications. Comparisons with other methods show the advantage of our method in terms of accuracy and robustness.

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