Local Noise Measurement in Real-Time Cardiac Cine MRI - A Random Matrix Approach

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Introduction: Accurate local noise assessment is important to evaluate the quality of a dynamic MR image series, as well as the performance of a specific imaging protocol when parallel imaging is used. National Electrical Manufacturers Association (NEMA) recommends assessing noise level from a noise-only image constructed from the subtraction of two images [1]. However, when it is applied to a dynamic image series with significant motion or contrast variations, a bias due to residual coherent structure may appear. We propose a generalization of NEMA’s recommendation to construct a stack of noise-only images using linear combinations of original images. The noise level can be assessed accurately from these noise-only images. When a dynamic image series bears temporal redundancy, some of the eigenimages of the temporal singular value decomposition (SVD) contain only noise. It has been shown that these noise-only eigenimages can be identified since their corresponding singular values follow the Marcenko-Pastur (MP) distribution of the random matrix theory (RMT) [2]:

\[ p(\lambda^2) = \frac{N}{2\pi\sigma^2\lambda^2M} \sqrt{\max(0, (\lambda_{\text{min}}^2 - \lambda^2)(\lambda^2 - \lambda_{\text{max}}^2))} \]

where \( \lambda_{\text{max, min}}^2 = \sigma^2 (1 \pm \sqrt{M/N})^2 \), \( N \) is a parameter proportional to the number of pixels per image, \( M \) is the number of noise-only images, \( \sigma \) is the mean noise standard deviation. All three above parameters can be determined by maximizing the statistical goodness-of-fit (GOF) between the smallest singular values and the MP-distribution. Once parameter \( M \) is determined, the last \( M \) eigenimages corresponding to the smallest \( M \) singular values are noise images. Local noise level of any region of interest (ROI) can be assessed from the average noise level across the stack of \( M \) noise images. We refer this method as RMT-fitting method in this abstract.

Methods: An experimental validation was performed in eighteen 256-frame real-time SSFP cine series with TSENSE acceleration factor 3, 4, and 5 in the same mid-ventricular short axis view acquired in six volunteers. Other imaging parameters were: 192 × 144 matrix, 8.0mm thick slice, flip angle 68 degrees, TE = 1.02 ms, TR = 2.36 ms, pixel bandwidth=1370 Hz/pixel. In each image series, the 32 pairs of images with the highest cross-correlation coefficient were selected. The subtraction images of these image pairs were evaluated visually to select the one showing the least amount of residual signal. Then NEMA’s noise measurement method was performed in this subtracted image [1] in a manually segmented ROI including the heart. The noise level in the same ROI was also measured by the RMT-fitting method. The relation of noise levels assessed by these two methods was analyzed using scatter plot and linear regression.

Results and Discussion: The average number of noise-only eigenimages found by the RMT-fitting method was 173 ± 28 across all image series. Fig. 1 shows an example in which the MP-distribution fit well to the distribution of the last 154 singular values of a real-time cardiac cine MR image series with 256 frames. In this example, 154 noise-only eigenimages could be constructed from corresponding eigenvectors. The GOF between the smallest singular values and the MP-distribution was evaluated in each image series using the Kolmogorov-Smirnov test. No significant deviation was observed (\( p \)-value > 0.99 in every image series). Fig. 2 compares the noise standard deviation measured by temporal subtraction and the RMT-fitting method. The \( R^2 \) value between the two measurement methods was 0.98. As expected, the RMT-fitting method found a slightly lower noise level than the temporal subtraction method (slope < 1), because the temporal subtraction method cannot completely differentiate spatial coherent structure from noise.

Conclusion: Our new method is a generalization of NEMA’s recommendation from the subtraction of two images to a linear combination of a dynamic image series. The in-vivo experiments demonstrate that this is an accurate method to assess local noise level in dynamic images.