Fast selection of phase encoding locations in parallel excitation

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Introduction
We propose a novel, fast method based on a hybrid version of Simultaneous Orthogonal Matching Pursuit(S-OMP)[1] to select sparse phase-encoding locations in a Echo-Volumar(VE) trajectory for parallel excitation pulse design with slice-selective subpulses [2]-[3]. In [2]-[3], the pulse length is dominated by the number of phase-encoding locations in the EV trajectory, so sparse phase-encoding is critical to generating a short RF pulse. This problem of enforcing sparsity in phase-encoding locations was presented as a convex optimization problem in a Second Order Cone Program(SOCP) form and solved in [3]. Unfortunately, typical SOCP routines with parallel solving are too slow to be computed in real-time, which may not be acceptable in many in-vivo MRI scans. We present a much faster greedy algorithm, Parallel-OMP(P-OMP), which will solve the same problem in a few seconds. We also show that the accuracy of our approach is very similar to that of SOCP in both the single and multiple coil cases.

Theory
In parallel excitation pulse design with slice-selective subpulses, the amplitudes of the subpulses are computed by solving $\arg \min_{b} \| d - A b \|_2$, where $d$ is the desired in-plane excitation pattern, $A = [S_{f_1} F, S_{f_2} F, ..., S_{f_r} F]$ is a stack of matrices where $S_i$ is a diagonal matrix of the i-th coil’s sensitivity pattern, and $F$ is a 2D-Fourier encoding matrix restricted to the support of $d$. The vector $b = [b_1; b_2; ...; b_r]$ is a vertically concatenated vector where $b_i$ is a vector composed of complex amplitudes of slice-selective pulses transmitted by the i-th coil and its element indices denote phase encoding locations. Each column of $F$ represents a candidate phase-encoding location. Therefore, enforcing sparsity in phase encoding locations can be viewed as selecting a minimal number of columns of $F$ such that a new matrix of selected columns, $F'$, can be used to form $A$ in place of $F$ to span $d$ or its close approximation. Our algorithm is an iterative procedure where at each step, we choose columns of $F$ that span a large portion of the residual of $d$, and then add those columns to $F'$, and update the residual by performing an orthogonal projection of $d$ onto $A = [S_{f_1}, ..., S_{f_r} F']$.

One important distinction that our problem makes from standard sparse approximation problems is that choosing one column of $F$ corresponds to $R$ columns in the approximation of $d$, because $A$ has $R$ columns associated with the selected column in $F$. Therefore, to assess how much of the residual is spanned by each candidate column in $F$, we experimented with three different approaches as in the algorithm layout on the left: P-OMP(Iteration), P-OMP(Projection) and P-OMP(projection).

P-OMP(Iteration): Let $F_i$ be i-th column of $F$. Let $j$ be a coil index. Set $F = \hat{F}$ an empty matrix. Set the initial residual $r_0 = d$. Set $k = 1$. Loop until the magnitude of the residual $\| r_k \|$ is sufficiently small. { - Compute cumulative correlation of $\hat{F}_i$ between columns in $F$ and $r_{k-1}$ - Pick columns whose cumulative correlation is bigger than a threshold - Add selected columns to $\hat{F}$ and do orthogonal projection of $d$ onto $\hat{A} = [S_{f_1}, ..., S_{f_k} F']$ - Set the new residual $r_k = d - \hat{A} \tilde{b}$ }

P-OMP(projection): do orthogonal projection of $d$ onto $[S_{f_1}, ..., S_{f_k} F']$ and compute the magnitude of projected vector.

Experiments and Discussion
To compare the performance between the convex optimization method[3] and our method, we measured the Normalized Root Mean Squared Error(NRMSE) of the approximated pattern with these two methods as we increment the number of phase encoding locations. Fig1 and Fig2 show the NRMSE measured in the computer simulation. On a computer with Intel Core2 Quad CPU 2.4Ghz, 4GB RAM and Matlab 7, we ran the above two algorithms to excite a uniform circular pattern of a radius 10.125cm with a single coil and with 8 coils. The field of excitation is 24cm by 24cm over a 64x64 sampling grid. As seen in the Figs.1 and 2, the NRMSE curves along the number of chosen phase-encoding locations show that our method shows compatible accuracy to the convex optimization. Also the Table 1 shows that our method runs much faster than the convex optimization on the same problem instance.

Fig 1. NRMSE in a single coil excitation Fig 2. NRMSE in 8 coils, parallel excitation

Conclusion
Our P-OMP-based method to enforce sparse phase-encoding locations in parallel excitation performs comparably with the SOCP solver in terms of accuracy. However, our greedy approach is significantly faster, and thus, is more desirable in applications where on-line computation of the RF pulse is crucial.


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