HYBRID MODELLING OF AXONAL MAGNETIC FIELDS FOR DIRECT MR NEURONAL DETECTION ESTIMATION

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Introduction

The aim of this study is to computationally estimate local magnetic fields from evoked axons to support previously reported experimental evidence of direct neuronal detection (DND) of axon firing using MRI [1,2]. The hypothesis tested is that the fields resulting from the electrical activity of axons can perturb the main \( B_o \) field of the scanner, thereby producing visible image artefacts so that direct imaging of brain functioning may be possible. In this work magnetic fields were calculated for a single axon using a novel hybrid model of the electro-physiological parameters of the human optic nerve, and a complete 3 dimensional solution of the differential form of Poisson’s equation.

Theory and Methods

The axon was modelled as connected segments of alternating active and passive regions. Axons in the human optic nerve are composed of myelinated fibres, and this myelination makes it possible for the axonal signals to travel longer distances and also enables higher conduction velocities compared to non-myelinated axons. The active conduction of ions in myelinated axons takes place at specified locations, known as the Nodes of Ranvier. In between these nodes long myelinated segments where passive conduction takes place. In our model, active and myelinated passive regions were simulated using the Hodgkin Huxley model and cable theory respectively [3, 4]. This hybrid model for the axons of the optic nerve is used for the first time for magnetic field estimation in this study. Figure 1a shows the equivalent electric circuit model of the axon which was implemented using the NEURON simulator [5]. A diameter of 1 micron was used for the axon which is appropriate for the human optic nerve. At this small diameter a stimulus in the range 0.03 – 0.2 nA input to NEURON caused an axonal action potential train, which is shown at three different points along the axon in Figure 1b as a function of time. The axon was divided into small compartments and the voltage distribution at each of these was calculated at time steps of 25 \( \mu \)s. The local magnetic field resulting from the spike train was then calculated using the magnetic vector potential obtained from a 3D solution of Poisson’s equation. The neuron was assumed to be present in a volume representing a typical voxel used for MRI DND. A 1mm cubic voxel was meshed with 11 grid points along each of the three spatial dimensions, with a mesh size of \( 100 \mu m \) along each axis. Using the voltage distribution \( V \) calculated from NEURON, the current density along the axonal \( z=0 \) axis in the centre of the x-y plane was calculated using 

\[
J_z = \frac{V}{R a^2} \quad \text{where } a \quad \text{and } R (=10\Omega) \text{ represent the axonal radius and resistance respectively. This current density then acted as a boundary condition for a finite difference solution of the Poisson equation,}
\]

\[
\nabla^2 A_z = -\mu_o J_z + q(\frac{\partial^2 A_z}{\partial z^2}) \quad \text{for the vector potential } A_z \quad \text{throughout the voxel, from which the axonal magnetic field could be obtained from } B = \nabla \times A \quad ,\text{ where } \mu_o \text{ is the permeability of free space and } \varepsilon \text{ the permittivity.}
\]

Results

The resulting \( B \) field values were in the range 2-25nT, with the field strongest at the axonal axis and decreasing with distance from the axon. This reduction in field strength with distance from the axon is relevant in MEG where the sensors are relatively far removed from the axon. However, for DND this is less relevant due to the spatial localization of the perturbations in the MR image, and therefore here the average \( B \) field within a voxel is a more relevant quantity. The \( B \) field components and total field (\( \sqrt{B_x^2 + B_y^2 + B_z^2} \)) averaged over the meshed voxel are shown in Figures 2a and 2b respectively as a function of time. Since the complete form of the Poisson equation was used for the vector potential calculation, propagation effects were also accounted for due to the addition of the time derivative. However, while it is possible that high frequency components of fast transients may produce a spectrum of wavelengths, for the problem space under consideration a quasi-static approach would be sufficient. The calculated field values are of the same order of magnitude as neuronal field measurements reported in previous DND studies [2].

Conclusion

In this study a realistic human optic nerve axon was modelled using a novel hybrid technique and 3D solution to the Poisson equation, to estimate axonal magnetic fields. The calculated field values were in the range of 2-25 nT which is commensurate with those reported in experimental studies, and so adds credence to their findings that direct detection of axonal magnetic fields is indeed possible using MRI. Such field values should also lie within the detection range of modern day MRI scanners. More realistic populations of neurons will be simulated in future to study any field cancellation that might occur, which could explain why a single axon produces field levels commensurate with a measured nerve containing millions of axons. Efforts are also underway to compare the results with other methods used for magnetic field estimations including Biot-Savart and Ampere laws.

References


Figure 1: a) Equivalent Electric circuit for the axon model used to simulate the trans-membrane potential. b) Potential distribution resulting from current stimulus at different points along the axon

Figure 2: a) Average Bx, By and Bz components of voxel field. b) Resulting average magnitude of voxel B field