Optimization of the encoding scheme for improved SNR efficiency in Vessel Encoded Pseudo-Continuous ASL

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Introduction:
In vascular territory imaging, the Hadamard type encoding[1-3] and other methods⁴ encode different vessels with different inversion efficiencies, such efficiencies are then used to decode the vascular territories. Though in theory the Hadamard encoding scheme will always give the optimal encoding efficiency⁵, in practice, given a various vascular shapes and locations, it is not always obvious how to construct such an encoding matrix, especially when more than 3 vessels are encoded. Here we introduce an encoding scheme to optimize the SNR efficiency in Vessel Encoded Pseudo-Continuous ASL with Hadamard type encoding.

Methods:
From collected in vivo data, we found the combination of three harmonic cosine components: $0.911\cos(2\pi x/\lambda) + 0.414\cos(2\pi - 2x/\lambda) + 0.089\cos(2\pi - 3x/\lambda) - 0.414$, could describe the VEPCASL tagging profile along the encoding direction with our implementation parameters, as shown in Figure 1, where $\lambda$ is the spatial period of the repeated inversion/non-inversion cycles, $x$ the distance from one arbitrary un-inverted position. For any number of source artefacts, if there are no constrains on $\lambda$, one can always achieve optimal encoding by increasing the encoding steps to find an encoding matrix with columns from a Hadamard matrix. The SNR efficiency for the $i_{th}$ vessel could be calculated with the formula [1]:

$$E_i = \frac{SNR_{encoded}}{SNR_{averaging}} = \frac{1}{\sqrt{N\sum_j |A_{i,j}|^2}},$$

where $N$ is the steps in one encoding cycle, $A^*$ is the pseudo-inverse of the encoding matrix, $j$ is the column number of $A^*$. In our simulation, randomly distributed vessel locations were used, and the optimal encoding steps are then calculated with different number of encoding steps using the criterion $\max_{i} \left(\min\left(E_i\right)\right)$, where $E_i$ is calculated using the formula above. Since in actual scan, control images are easy to collect without big error, one of the encoding steps is set to in control condition. The mean lowest encoding efficiency of vessels and their standard deviation is shown in Figure 2 A&B. At least $n+1$ encoding steps are required for encoding and decoding vascular territory information from $n$ vessels, since the ASL signal inherently required repeated data collection, it is convenient to increase the encoding steps in each cycle while keeping the total acquire repetitions the same without any extra SNR efficiency penalty unless some encoding steps with small relative tagging efficiencies are used. Generally, to separate $n$ vessels, at least $2^{\lceil\log_2(n+1)\rceil}$ encoding steps are required to achieve such optimal SNR efficiency [3]. Under such condition, repeating previous fully encoded steps with the relative tagging efficiency sign reversed in the same encoding cycle would not reduce the SNR efficiency, but give the benefits that if control images are acquired, then images with all the vessels inverted will also be collected, which ensures that the relative inversion efficiency of each vessel in each step could be estimated [3].

Results:
For 3 vessels, the SNR efficiencies decreased and then increased when the encoding steps increased from 4 to 8. When the encoding step is 4 or 8, the highest efficiency is achieved since a Hadamard-type matrix can always be formed for 3 non-colinear vessels. For the 4 vessel case, the efficiency reaches its first peak when 8 encoding steps are used. As the encoding step number increases, the optimization becomes computationally intensive, and some of the simulations failed to reach the optimal solution within the iteration limits, which contributed to the larger deviation, however, the SNR efficiency does improve when using $2^m \cdot 2^{\lceil\log_2(n+1)\rceil}$ encoding steps, where $m = 0,1,2,...$

Discussion:
The fitted tagging profile function from the in vivo data could give a good prediction of the encoding SNR efficiency, thus provide optimal encoding schemes based on optimal solution search methods. Though the best SNR efficiency could be achieved with a small number of encoding steps in ideal case, in practice, the smallest spatial periods of the tagging profile, $\lambda$, will be constrained by the gradient amplitude limitation. However, the encoding scheme with optimal SNR efficiency under these constrains could always be achieved.

Reference: