Optimizing the diffusion weighting gradients for diffusion-kurtosis imaging

D. H. J. Poot1, A. J. den Dekker2, M. Verhoeye3, I. Blockx4, J. Van Audekerke5, A. Van Der Linden3, and J. Sijbers4
1Visionlab, University of Antwerp, Antwerp, Belgium, 2Delft Center for Systems and Control, TUDelft, Delft, Netherlands, 3Bio-Imaging Lab, University of Antwerp, Antwerp, Belgium

Introduction
Diffusion kurtosis imaging is a new method to estimate the non gaussianity of the diffusion process with diffusion weighted MR images (DWIs) [1]. The precision with which kurtosis parameters can be estimated from the acquired DWI depends on the direction and strength of the diffusion weighting gradients. In this work, the experimental design is optimized by minimizing the Cramér-Rao-lower-bound (CRLB) [2] as function of the direction and strength of the diffusion weighting gradients of the kurtosis measures of interest.

Methods
The magnitude of the Rician distributed diffusion weighted images (DWIs) is modeled by

$$A_i(\theta_p, b) = A_i \exp \left( \frac{1}{6} \left( \frac{b_i b_j b_k}{b_i b_j} \right) b_j b_k b_l b_m K_{ijk} \right).$$

(1)

with \(b = B_1\) is the \(i^{th}\) diffusion weighting gradient and \(\theta_p\) a parameter vector containing 22 elements. \(\theta_p\) models \(A_i\) and the fully symmetric rank 3 tensors \(D\) (diffusion) and \(K\) (kurtosis) of order 2 and 4, respectively. At least 22 DWIs, each with suitably different diffusion weighting gradient settings, need to be recorded. Often many more DWIs are recorded to obtain more precise kurtosis estimates. To optimize the gradient directions and strengths for the DWIs, the CRLB, which is the inverse of the Fisher information matrix \(I(\theta_p, B)\), is computed for \(M\) parameter vectors \(\theta_p\):

$$B = \arg \min_B \sum_{p=1}^{M} f(\theta_p, B).$$

(2)

The function \(f\) in Eq. (2) provides a scalar measure of the CRLB of each \(\theta_p\). The actual measure chosen might depend on the goal of the experiment, such as precise estimation of the mean or the radial kurtosis. This measure is minimized by simulated annealing, which is chosen because it can be efficiently evaluated, handle the large number of degrees of freedom (3 x #DWI) and the many local minima present in the optimization problem. Simulated annealing, in general, cannot guarantee convergence to the global maximum in a limited amount of time. However, a close to optimal set of gradients suffices for the recording of the diffusion weighted images.

Simulations
Simulation experiments were run with a set of 120 simulated diffusion tensors with small kurtosis (kurtosis anisotropy was approximately 0.1). As reference gradient set, a set with 5 non diffusion weighted images (B0 images) and 28 diffusion weighting directions at each of 5 evenly spaced gradient strengths (b-values), with optimized base b-value, was used. The directions of one half of 56 symmetric, evenly distributed, points on the unit sphere were used as the gradient directions. The optimized gradient set also consisted of 145 gradient direction - b-value combinations, which were simultaneously optimized in both direction and b-value.

The ratio of the CRLB of the radial kurtosis of the optimized versus the un-optimized gradient set was 0.72. This indicates the importance of the optimization of the gradients, since in order to obtain the same reduction of the variance with the un-optimized gradient set, almost 40% extra diffusion weighted images would be needed.

A further validation of the optimized gradient set was performed with realistic data. For this, a dataset with diffusion and kurtosis parameters was estimated from a set of measured DWIs. See figure 1(a) for an image displaying the radial kurtosis of this dataset. The estimated diffusion and kurtosis parameters were then used to simulate DWIs with both gradient sets, where noise was added to obtain realistic Rician distributed DWIs. Figure 1(b) and 1(c) show the radial kurtosis estimates obtained from these simulated DWIs, for the un-optimized and optimized gradient sets, respectively. Note that the variance in image 1(c), obtained with the optimized gradients, is significantly reduced compared to image 1(b), obtained with the un-optimized gradients, which indicates the better performance of the optimized gradient set.

Conclusions
Optimization of the gradient set is important for Diffusion Kurtosis imaging, since the optimization substantially increases the precision with which the kurtosis measures can be estimated from a given number of DWIs. This increase in precision could also be used to reduce scan time by reducing the number of DWIs that are recorded.

References