Improved field map estimation in the presence of multiple spectral components

D. Hernando[1], P. Kellman[2], J. Haldar[1], and Z-P. Liang[1]

[1]Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, United States, [2]Laboratory of Cardiac Energetics, National Heart, Lung and Blood Institute, National Institutes of Health, Bethesda, MD, United States

INTRODUCTION

Estimation of the B_0 field inhomogeneity map is an important problem in MRI, as it allows, e.g., robust water/fat separation, improved shimming, and correction of EPI and spiral acquisitions. A common approach for measuring the field map is based on the phase evolution of several images acquired at different TEs. The estimated field map is typically regularized by imposing spatial smoothness [1,2]. However, this approach is not robust in the presence of severe field inhomogeneity (e.g., in abdominal or cardiac imaging, which have significant susceptibility changes), due to the nonconvexity of the corresponding optimization problem [3,4]. Here we introduce a more robust method for regularized field map estimation in the presence of multiple spectral components.

METHODS

Consider an acquisition consisting of N images $I_{(x,y)}(t_n)$ obtained at different TEs, t_n, $n=1,\ldots,N$. For simplicity, let us focus on the case with two spectral components (water and fat). At a given voxel, the signal can be well modeled as $s(t_n;\, \rho_W, \rho_F, f_B) = [\rho_W + \rho_F \exp(i2\pi t_n f_F)]\exp(i2\pi t_n f_B)$, where ρ_W and ρ_F are the water and fat amplitudes, respectively, f_F is the (known) fat chemical shift, and f_B is the frequency shift due to field inhomogeneity. Assuming Gaussian noise, maximum-likelihood (ML) estimation of (ρ_W, ρ_F, f_B) reduces to minimizing $R_{(x,y)}(\rho_W, \rho_F, f_B) = ||\mathbf{I}_{(x,y)} - \mathbf{s}(\rho_W, \rho_F, f_B)||^2$, where $\mathbf{I}_{(x,y)} = [I_{(x,y)}(t_1)\ \ldots\ I_{(x,y)}(t_N)]^T$ is the vector of measured data at voxel (x,y), and $\mathbf{s}(\rho_W, \rho_F, f_B)=[s(t_1;\rho_W, \rho_F, f_B)\ \ldots\ s(t_N;\rho_W, \rho_F, f_B)]^T$. The variable projection (VARPRO) method allows us to remove the linear variables, resulting in a new cost function $R_{(x,y)}(f_B) = ||\mathbf{I}_{(x,y)} - \mathbf{\Phi}(f_B)\mathbf{\Phi}^{\dagger}(f_B)\,\mathbf{I}_{(x,y)}||^2$, where $\mathbf{\Phi}(f_B)$ is the $Nx2$ matrix with rows $[\exp(i2\pi t_n f_B)\quad \exp(i2\pi t_n(f_B + f_F))]$, for $n=1,\ldots,N$, and † denotes pseudoinverse. Minimizing $R_{(x,y)}(f_B)$ is a one-dimensional problem at each voxel [4].

The key aspect of field map estimation is spatial regularization. For this purpose, the effectiveness of imposing a Markov Random Field (MRF) smoothing prior has been studied in [4]. However, field map estimation subject to an MRF prior is a high-dimensional problem with multiple local optima, and fast algorithms such as iterated conditional modes (ICM) guarantee only local convergence, whereas stochastic algorithms which guarantee asymptotic global convergence are extremely slow. The proposed method is designed to overcome these limitations by noting that ICM will converge to the global optimum as long as the initial guess is close enough to the global optimum. Furthermore, a good approximation to most field maps can be made as a combination of smooth basis functions $\varphi_j(x,y)$ (e.g., low-frequency sinusoids). The VARPRO formulation allows us to express the approximated field map as $f_{B,\text{smooth}}(x,y) = \sum_j c_j \varphi_j(x,y)$ and estimate the coefficients c_j directly from the data (instead of voxel-by-voxel estimation with subsequent low-pass filtering) by minimizing the sum of the residuals at every location $\sum_{x,y} R_{(x,y)}(f_{B,smooth}(x,y))$ (e.g., by a coordinate descent method). This is equivalent to constrained ML estimation, and can be done efficiently by pre-computing $R_{(x,y)}(f_B)$ at each voxel, on a grid of f_B values [4]. Subsequently, $f_{B,\text{smooth}}(x,y)$ is used as initial guess for ICM, which will provide the final field map estimate. This final estimate may contain more rapid field variation than that allowed by the smooth approximation, thus accommodating susceptibility effects or regions of high field inhomogeneity within the magnet.

RESULTS

Cardiac data were acquired using a multi-echo GRE sequence on a Siemens ESPREE 1.5T scanner using four channels, with TEs *1.58*, *3.91*, *6.24*, and *8.57* ms. Figure 1 shows results from field map estimation and the corresponding water/fat separation, and illustrates the advantages of jointly estimating the field map and imposing smoothness, instead of imposing smoothness *a posteriori*.

The robustness of the estimation process is due to the ability of the initial step (estimating $f_{B,smooth}$) to capture the main features of the field map variation, and therefore encourage the final estimate to fall in the correct "valley" of the residual $R_{(x,y)}(f_B)$ at each location. Note that estimation of $f_{B,smooth}$ is itself a nonconvex problem, but one defined in a much lower-dimensional space such that many voxels are updated simultaneously. Thus, it has the ability to avoid local minima in which voxel-by-voxel updating would get trapped. Moreover, despite the extreme smoothness of $f_{B,smooth}$, the final ICM estimate is still able to recover sharper field variations if they are present in the data.

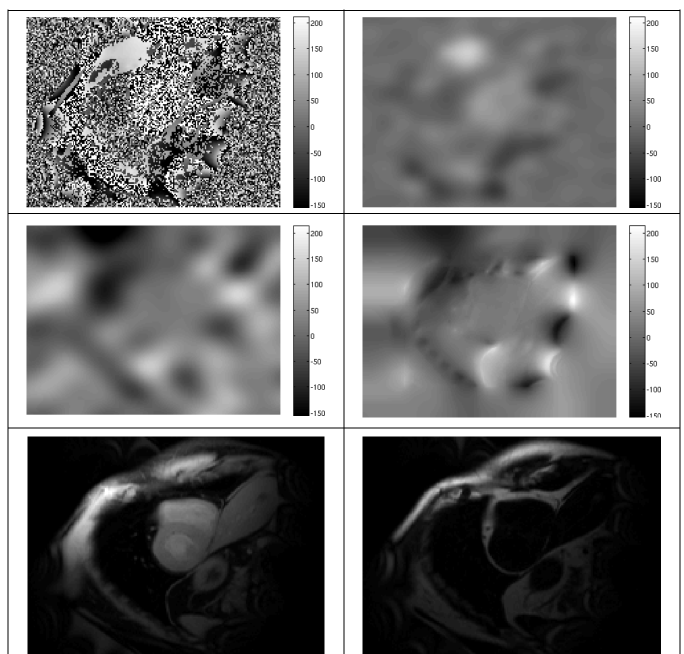

Figure 1. (Top left) Voxel-by-voxel ML field map estimate; (Top right) Low-pass filtered ML estimate, not able to correct the errors introduced by voxel-by-voxel processing; (Center left) Proposed method: initial field map expressed as a combination of low-frequency sinusoids ($f_{B,smooth}$); (Center right) Final field map obtained by ICM initialized with $f_{B,smooth}$; (Bottom left) Resulting water image; (Bottom right) Resulting fat image.

CONCLUSION

This work presents a two-step procedure for B_0 field map estimation in the presence of multiple spectral components. This method produces better results than existing techniques by formulating the regularized estimation of the complete field map as a joint problem, which is solved using an improved optimization algorithm.

REFERENCES

[1] S. Reeder et al., Magn Reson Med 2005, 54:636-644.
[2] J. Fessler et al., Proc. ISBI 2006, pp. 706-709.
[3] H. Yu et al., Magn Reson Med 2005,54:1032-1039.
[4] D. Hernando et al., Magn Reson Med, *accepted*.