Estimation of water/fat images, $B_0$ field map and $T_2^*$ map using VARPRO

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INTRODUCTION

Conventional water/fat separation methods based on Dixon-type acquisitions do not account for the $T_2^*$ decay effect, which leads to underestimated intensities of the water/fat components in regions where the decay is significant [1,2]. It is, therefore, desirable to estimate the $T_2^*$ in order to improve the quality of water/fat decomposition. Furthermore, the $T_2^*$ map itself is of clinical value, e.g., for the diagnosis of iron overload [2,3]. Here we propose a novel method, based on the variable projection (VARPRO) formulation [4], for robust and efficient estimation of water/fat images, field map and $T_2^*$ map.

METHODS

In a Dixon acquisition with $N$ images acquired at echo times $TE=t_n, n=1,...,N$, the signal at a given voxel can be modeled as

$$s(\rho_w, \rho_f, f_B, R_2^*) = \begin{cases} e^{i\omega_{B0}t_n + i\phi_{B0}} e^{-\rho_w t_n} & \text{if } f_B = \text{real}, \\ e^{i\omega_{B0}t_n + i\phi_{B0}} e^{-\rho_f t_n} & \text{if } f_B = \text{imaginary} \end{cases}$$

where $s = [s(t_1), s(t_2), ..., s(t_N)]^T$ is the signal vector, $\rho_w$ and $\rho_f$ are the complex-valued water and fat amplitudes, respectively, $f_B$ is the fat chemical shift (e.g., -220 Hz at 1.5 T), $f_B$ is the frequency shift due to $B_0$ field inhomogeneity, and $R_2^* = 1/T_2^*$. Assuming white Gaussian noise, the maximum-likelihood estimation of $(\rho_w, \rho_f, f_B, R_2^*)$ is a nonlinear least-squares problem (i.e., minimizing $L(\rho_w, \rho_f, f_B, R_2^*) = \|s_{\text{true}} - \Phi(f_B, R_2^*)\rho\|^2$).

However, using VARPRO, this problem is equivalent to minimizing $L(\rho_w, \rho_f, R_2^*) = \|s_{\text{true}} - \Phi(f_B, R_2^*)\rho\|^2$ (for $f_B = \text{real}$). This minimization can be performed by evaluating $L(f_B, R_2^*)$ on a 2D grid and directly picking the minimum.

The VARPRO formulation has several desirable features: it avoids the local convergence of iterative search algorithms. It also allows the use of prior constraints on $R_2^*$. Finally, it allows effective field map regularization (which is the most challenging aspect of the whole estimation process).

One drawback of this approach is its computational cost, since evaluation of $L(f_B, R_2^*)$ on a 2D grid is relatively time-consuming. However, Cramer-Rao bound (CRB) analysis of the signal model in Eq. (1) indicates that, for an efficient estimator, the estimates for $f_B$ and $R_2^*$ are uncorrelated (i.e., the corresponding cross-term in the CRB matrix is zero for all the combinations of parameter values we have tried). This observation leads to the following efficient decoupled VARPRO method: 1) Estimate the regularized field map using the original VARPRO method (assuming no decay). 2) Given the estimated field map, obtain $R_2^*$ at each voxel using VARPRO. 3) Given the field map and $R_2^*$ map, estimate $\rho_w$ and $\rho_f$ at each voxel by solving the corresponding linear problem in Eq. (1). This method requires two 1D searches, instead of one 2D search, resulting in notable computational savings.

RESULTS

Cardiac images were acquired using a multi-echo GRE sequence on a Siemens Espree 1.5T. A total of 13 TEs were acquired to provide a “gold standard” field map and $R_2^*$ map, produced from 13 TEs. The estimated $R_2^*$ map itself is of clinical value, e.g., for the diagnosis of iron overload [2,3]. Here we propose a novel method, based on the variable projection (VARPRO) formulation [4], for robust and efficient estimation of water/fat images, field map and $T_2^*$ map.

REFERENCES


Figure 1. (a)-(b) Water/fat decomposition estimated from 13 TEs. (c)-(d) “Gold-standard” field map and $R_2^*$ map, jointly estimated using all 13 TEs. The estimated $R_2^*$ is shown for the ROI displayed within the water image. (e)-(f) Field map and $R_2^*$ map, jointly estimated using 6 TEs. (g)-(h) Field map and $R_2^*$ map, estimated using decoupled VARPRO with the same 6 TEs. Note the high-quality of the estimates obtained from 6 TEs. Furthermore, the joint and decoupled estimation of field map and $R_2^*$ map produce very similar results (the relative difference in $R_2^*$ over the ROI was 1.2%). However, joint estimation required 15,000 evaluations of $L(f_B, R_2^*)$ per voxel, whereas decoupled estimation required 350.

Equation (1) assumes a unique $R_2^*$ value per voxel. In voxels where both water and fat are present, this estimation will provide an “effective” $R_2^*$ estimate [2]. VARPRO can be easily adapted to estimate two distinct decay constants, although this will result in noisier estimates and increased computational times. Note also that a common method for $R_2^*$ estimation is to assume a single spectral component and fit an exponential to the signal magnitude. This method produces significantly worse results than the method proposed here in the presence of several spectral components.

CONCLUSION

This work presents a method for estimating $B_0$- and $T_2^*$-maps along with water/fat images from Dixon acquisitions, by extending a recently proposed variable projection method. This method provides accurate estimates regardless of the nonconvexity of the corresponding estimation problem.